

**Instructions:** Do your own work. You may consult your class notes and the course text. Do not consult other sources. Do not discuss generalities or specifics of the exam with anyone except me. This exam is due by Friday, May 14 at 2 pm (the end of the scheduled final exam period for the course).

This exam is a tool to help me (and you) assess how well you are learning the course material. As such, you should report enough written detail for me to understand how you are thinking about each problem.

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1. Consider a function  $f(z) = \frac{p(z)}{q(z)}$  where  $p$  and  $q$  are analytic at  $z_0$ ,  $p(z_0) \neq 0$ , and  $q$  has a zero of order 2 at  $z_0$ .
  - (a) Find an expression for the residue of  $f$  at  $z_0$  in terms of  $p$ ,  $q$ , and derivatives of these functions. (10 points)
  - (b) Use your result to compute the residue of  $f(z) = \frac{z+1}{\sin^2 z}$  at  $z_0 = 0$ . (6 points)
  
2. Evaluate  $\oint_C \frac{25z^3}{(z+1)^2(z-2i)} dz$  where  $C$  is the circle of radius 4 centered at the origin. (16 points)
  
3. Evaluate  $\int_C \frac{\sin z}{1+e^z} dz$  where  $C$  is the circle of radius 2 centered at  $2i$ . (16 points)
  
4. Evaluate  $\int_C \frac{z^{99}}{1+z^{100}} dz$  where  $C$  is the circle of radius 2 centered at the origin. (16 points)
  
5. Evaluate  $\int_0^\infty \frac{1}{1+z^8} dz$  using complex techniques with each of the following contours.
  - (a) the “standard” contour consisting of the segment  $[-R, R]$  and the semicircle  $C_R$  (14 points)
  - (b) a more clever choice of contour that encloses only one relevant pole (8 points)
  
6. Suppose the Math Club exists and it publishes a monthly newsletter for math students. Write an article that describes complex analysis for this newsletter. Consider your target audience to be math students who have completed the calculus sequence and linear algebra but who have *not* taken complex analysis. Your goal is to give those students guidance in making an informed decision about taking a course in complex analysis. Focus on aspects of complex analysis that you think are important or interesting. You do not need to summarize every idea that we have covered this semester. Your opinion of complex analysis does not need to be positive. I will evaluate your essay in terms of the following criteria: theme and coherence; mathematical precision and accuracy; understanding and insight; technical aspects such as grammar and spelling. (14 points)