Name ______________________________

MATH 352 Complex Analysis Spring 2010 Exam #4

Instructions: Do your own work. You may consult your class notes and the course text. Do not consult other sources. Do not discuss generalities or specifics of the exam with anyone except me. This exam is due in class on Thursday, April 22.

This exam is a tool to help me (and you) assess how well you are learning the course material. As such, you should report enough written detail for me to understand how you are thinking about each problem.

---

1. Find the 101^{st} derivative of $z^2 e^{-5z^3}$ for $z = 0$ (without using computing technology). (15 points)

2. Compute the Taylor series for $\operatorname{Log} z$ based at $z_0 = 1$ and give the region of convergence for this series. (15 points)

3. For each of the following regions, find the Taylor or Laurent series expansion for the function $f(z) = \dfrac{1}{1+z^2} + \dfrac{1}{3-z}$ that is valid for that region. (10 points each)

   (a) $|z| < 1$ (b) $1 < |z| < 3$ (c) $3 < |z|$

4. We have previously determined the Laurent series for the function $f(z) = \dfrac{1}{4z - z^2}$ that converges for $0 < |z| < 4$. We did this by rewriting the function so that we could take advantage of the geometric series result. As usual, let b_1 be the coefficient on the $1/z$ term. Compute b_1 directly and compare this with the value of b_1 we got in our previous work. (15 points)

5. Your friend makes the following argument:

   Consider the function defined by
   $$f(z) = \cdots + \frac{1}{z^3} + \frac{1}{z^2} + \frac{1}{z} + 1 + z + z^2 + z^3 + \cdots$$
   Note that
   $$1 + \frac{1}{z} + \frac{1}{z^2} + \frac{1}{z^3} + \cdots = \frac{1}{1 - 1/z} = -\frac{z}{1-z}.$$
   Also note that
   $$z + z^2 + z^3 + \cdots = z(1 + z + z^2 + \cdots) = z\frac{1}{1-z} = \frac{z}{1-z}.$$
   Therefore,
   $$f(z) = -\frac{z}{1-z} + \frac{z}{1-z} = 0.$$
   In other words, f is the zero function.

   Is your friend's argument correct? If not, explain any flaws in the argument. Is the conclusion correct? If f is not the zero function, determine what is true about f as a function. (15 points)

6. Prove the following: If f is entire and $\displaystyle\lim_{z\to\infty} \frac{f(z)}{z} = 0$, then f is constant. (10 points)