Computing the mean of a two-dice sum distribution

Earlier this week, three groups collected 500 values for the sum on a pair of dice. Here is the data distribution with values arranged in increasing order:

2 2 2 2 2 2 2 2 2 3 3 3 3 3 3 3 3 3 3 3 3 3 . . . 11 11 11 11 11 11 11 11 11 11 12 12 12 12 12 12 12 12 12

OK, I left out a bunch of values in the middle, but you get the idea. Each possible outcome (2 through 12) is repeated many times. We can record a count for each outcome and a proportion for each outcome. Here's those numbers:

Outcome	Count	Proportion
2	6	$\frac{6}{500} = 0.012$
3	29	$\frac{29}{500} = 0.058$
4	45	$\frac{45}{500} = 0.090$
5	58	$\frac{58}{500} = 0.116$
6	70	$\frac{70}{500} = 0.140$
7	94	$\frac{94}{500} = 0.188$
8	63	$\frac{63}{500} = 0.126$
9	54	$\frac{54}{500} = 0.108$
10	42	$\frac{42}{500} = 0.084$
11	23	$\frac{23}{500} = 0.046$
12	16	$\frac{16}{500} = 0.032$
Total	500	$\frac{500}{500} = 1.000$

Now let's examine how to compute the mean of this distribution. To do this, we go back to the listing of all the values in the distribution and compute

$$\bar{x} = \frac{2 + 2 + 2 + 2 + \ldots + 12 + 12 + 12 + 12}{500}$$

The numerator is a sum of all 500 values. We can reorganize the arithmetic to get a different way of thinking about the mean. Using the counts from the table, we can replace repeated addition of the same value with multiplication and write this as

$$\bar{x} = \frac{(2 \times 6) + (3 \times 29) + (4 \times 45) + (5 \times 58) + \cdots + (10 \times 42) + (11 \times 23) + (12 \times 16)}{500}$$

Using rules for adding fractions, we can rewrite this as

$$\bar{x} = \frac{2 \times 6}{500} + \frac{3 \times 29}{500} + \frac{4 \times 45}{500} + \frac{5 \times 58}{500} + \cdots + \frac{10 \times 42}{500} + \frac{11 \times 23}{500} + \frac{12 \times 16}{500}$$

Finally, we can reorganize the multiplications (using associativity) to get

$$\bar{x} = 2 \times \frac{6}{500} + 3 \times \frac{29}{500} + 4 \times \frac{45}{500} + 5 \times \frac{58}{500} + \cdots + 10 \times \frac{42}{500} + 11 \times \frac{23}{500} + 12 \times \frac{16}{500}$$

In this new form, we compute the mean by first multiplying each outcome by its proportion in the distribution and then summing those results. In this case, the mean is $\bar{x} = 7.004$.

What's the point of all this? Well, for a (discrete) random variable, we don't have a list of values or a list of counts. We only have a probability for each outcome. But that is enough to compute a mean using the reorganized arithmetic. That is, the mean of a discrete random variable is computed by first multiplying each outcome by the probability for that outcome and then adding up those results.