**Probability of at least two people sharing a birthday in a group of size $n$**

Consider a group of $n$ people and ask for the probability that at least two share a birthday. Let $A$ be the event of at least two sharing a birthday so $A^c$ is the event of no shared birthdays. We can determine $P(A^c)$ directly as follows. Think of $A^c$ as the event

"Any birthday for first person" AND "Any remaining birthday for second person" AND ... AND "Any remaining birthday for last person"

In a group of people chosen at random, the individual events are independent so

$$P(A^c) = P\begin{pmatrix}\text{“Any b-day for} \\ \text{first person”}\end{pmatrix} \times P\begin{pmatrix}\text{“Any remaining b-day} \\ \text{for second person”}\end{pmatrix} \times \cdots \times P\begin{pmatrix}\text{“Any remaining b-day} \\ \text{for last person”}\end{pmatrix}$$

The individual probabilities are easy to compute.* For the first person, there are 365 options out of 365 possibilities. For the second person, there are 364 options out of 365 possibilities. In this pattern, there are $365 - n + 1$ options out of 365 possibilities for the last person. We thus have

$$P(A^c) = \frac{365}{365} \times \frac{364}{365} \times \frac{363}{365} \times \cdots \times \frac{365 - n + 1}{365}$$

which we can rewrite as

$$P(A^c) = \frac{365 \times 364 \times 363 \times \cdots \times (365 - n + 1)}{365^n}.$$

Values for $P(A^c)$ and $P(A) = 1 - P(A^c)$ are given in the table below for various values of $n$.

| $n$ | $P(A^c)$ | $P(A)$ |
|---|---|---|
| 4 | 0.9836 | 0.0164 |
| 8 | 0.9257 | 0.0743 |
| 12 | 0.8330 | 0.1670 |
| 16 | 0.7164 | 0.2836 |
| 20 | 0.5886 | 0.4114 |
| 24 | 0.4617 | 0.5383 |
| 28 | 0.3455 | 0.6545 |
| 32 | 0.2467 | 0.7533 |
| 36 | 0.1678 | 0.8382 |
| 40 | 0.1088 | 0.8912 |
| 44 | 0.0671 | 0.9329 |
| 48 | 0.0394 | 0.9606 |
| 52 | 0.0220 | 0.9780 |
| 56 | 0.0117 | 0.9883 |
| 60 | 0.0059 | 0.9941 |

*Here, we will make some assumptions in our probability model. First, we are ignoring leap years by assuming all years have 365 days. Second, we are assuming that birthdays are uniformly distributed throughout the year.