Name ______________________________

MATH 301 Differential Equations Fall 2006 Exam #2

Instructions: You can work on the problems in any order. Please use just one side of each page and clearly number the problems. You do not need to write answers on the question sheet.

This exam is a tool to help me (and you) assess how well you are learning the course material. As such, you should report enough written detail for me to understand how you are thinking about each problem.

You can use integration aids such as a table of integrals.

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1. Determine if
$$y(t) = c_1 \frac{1}{t^2} + c_2 t^3 + t^3 \ln t$$
is the general solution of the differential equation
$$t^2 y'' - 6y = 5t^3$$
for the interval $(0, \infty)$. *You do not need to show where this formula for $y(t)$ comes from.* (16 points)

2. Consider the initial-value problem
$$t^2 y'' - 2ty' + 2y = 0, \quad y(0) = 0, \quad y'(0) = 0.$$
   (a) Show that $y(t) = Ct^2$ satisfies this initial-value problem for every choice of the constant C. (6 points)
   (b) Explain how the fact in (a) is consistent with the Existence-Uniqueness Theorem. (6 points)

3. For each of the following, find the general solution of the given differential equation. Express results in terms of real-valued functions. (12 points each)
   (a) $y''(t) - 6y'(t) + 4y(t) = 8t^2$
   (b) $y'' - 6y' + 14y = 0$
   (c) $y'' - \frac{2}{t}y' + \frac{2}{t^2}y = t^{5/2}$ for $0 < t < \infty$
   Hint: The complementary solution is $y_c(t) = c_1 t + c_2 t^2$.
   (d) $t^2 y''(t) - ty'(t) - 3y(t) = 0$ for $0 < t < \infty$
   Hint: Look for solutions in the form t^r where r is constant.

4. An object of mass m hangs on a spring with spring constant k. Damping forces are negligible. At $t = 0$, the object is at rest in the equilbrium position and an external force is turned on. The external force is initally at some positive value and then decays exponentially in time.
   (a) Set up an initial-value problem to model this situation. (10 points)
   (b) Solve the initial-value problem you set up in (a). (10 points)
   (c) For large values of t (in comparison with $1/r$), the motion of the object should be approximately sinusiodal. Find the amplitude of this sinusoidal motion. (4 points)