

**Instructions:** You can work on the problems in any order. Please use just one side of each page and clearly number the problems. You do not need to write answers on the question sheet.

This exam is a tool to help me (and you) assess how well you are learning the course material. As such, you should report enough written detail for me to understand how you are thinking about each problem. (100 points total)

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1. Solve each of the following equations. (4 points each)

(a)  $4e^{5k} = 3$

(b)  $5 \ln(y^2 + 2) = 14$

2. For this problem, do not use your calculator. (4 points each)

(a) Make a plot of the function  $f(x) = \sin x$  showing at least one full cycle starting with  $x = 0$ . Include relevant scales on each axis.

(b) Make a plot of the function  $f(x) = 3 \sin(2\pi x)$  showing at least one full cycle starting with  $x = 0$ . Include relevant scales on each axis.

3. Use a table of input/output pairs to make a conjecture about the limit  $\lim_{x \rightarrow 16} \frac{x^{1/4} - 2}{x - 16}$ . Give enough evidence to conjecture the limit to 3 significant digits. (8 points)

4. For each of the following, evaluate the limit (or conclude “does not exist”) using techniques that give an exact result if possible. Show enough details to make your methods clear to a reader. (7 points each)

(a)  $\lim_{x \rightarrow 3} \frac{\sqrt{x + 13} - 4}{x - 3}$

(c)  $\lim_{t \rightarrow 2} \frac{t^3 + 8}{t + 2}$

(b)  $\lim_{y \rightarrow 0} \frac{\sin(4y)}{y}$

(d)  $\lim_{x \rightarrow -3} \frac{9 - x^2}{x + 3}$

5. (a) Explain the distinction between  $\lim_{x \rightarrow c} f(x)$  and  $f(c)$ . (4 points)
- (b) Give an example of a function  $f(x)$  and a value  $c$  so that  $\lim_{x \rightarrow c} f(x)$  and  $f(c)$  are equal. (3 points)
- (c) Give an example of a function  $f(x)$  and a value  $c$  so that  $\lim_{x \rightarrow c} f(x)$  and  $f(c)$  are *not* equal. (3 points)

6. Consider the function  $f(x) = \begin{cases} x^2 - 5 & \text{if } x < 4, \\ 9 & \text{if } x = 4 \\ 4x + 10 & \text{if } x > 4. \end{cases}$

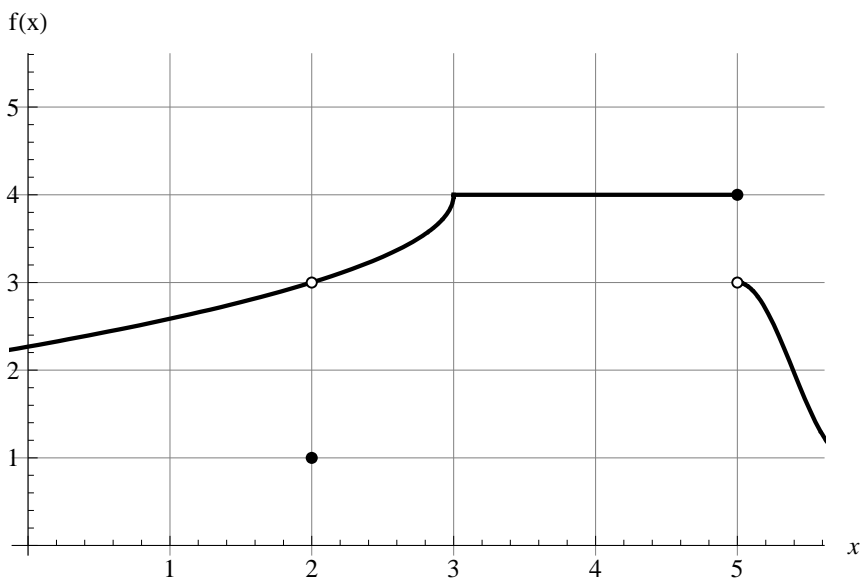
(a) Analyze the one-sided limits  $\lim_{x \rightarrow 4^-} f(x)$  and  $\lim_{x \rightarrow 4^+} f(x)$ . (8 points)

(b) Based on your results from (a), reach a conclusion about the (two-sided) limit  $\lim_{x \rightarrow 4} f(x)$ . (4 points)

7. Use the Sandwich Theorem to prove  $\lim_{x \rightarrow \infty} \frac{\cos x}{x} = 0$ . Hint: You can use the fact that  $-1 \leq \cos x \leq 1$  for  $x > 0$ . (9 points)

8. Evaluate  $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$  for the function  $f(x) = x^2$  and the value  $x = 5$ . (9 points)

9. Here's a plot of a function  $f$ :



(a) Give the value (or “does not exist”) for each of the following: (4 points)

i.  $f(2)$                       ii.  $\lim_{x \rightarrow 2^-} f(x)$                       iii.  $\lim_{x \rightarrow 2^+} f(x)$                       iv.  $\lim_{x \rightarrow 2} f(x)$

(b) Give the value (or “does not exist”) for each of the following: (4 points)

i.  $f(5)$                       ii.  $\lim_{x \rightarrow 5^-} f(x)$                       iii.  $\lim_{x \rightarrow 5^+} f(x)$                       iv.  $\lim_{x \rightarrow 5} f(x)$