Name ____________________

MATH 180 | Calculus and Analytic Geometry I | Spring 2009 | Project #2

Instructions

You should submit a carefully written report addressing the problems given below. You are encouraged to discuss ideas with others for this project. If you do work with others, you must still write your report independently.

Use the writing conventions given in *Some notes on writing in mathematics*. You should include enough detail so that a reader can follow your reasoning and reconstruct your work. All graphs should be done carefully on graph paper or using appropriate technology.

The project is due in class on Friday March 13.

Derivatives are often used in modeling real-world phenomena. An example is given in the accompanying paper "Reduction of HIV Concentration during Acute Infection: Independence from a Specific Immune Response", Andrew N. Phillips, *Science*, New Series, Vol. 271, No. 5248 (Jan. 26, 1996), pp. 497-499.

The model in the Phillips paper uses *differential equations*. A differential equation is an equation that involves an unknown function and its derivatives (first derivative or higher). Broadly speaking, there are three steps in modeling a real-world phenomenon with differential equations:

- formulate differential equations that reflect important aspects of the phenomenon
- analyze the differential equations to determine solutions or properties of solutions
- compare results from the analysis with data from experiment or observation

For this project, we will focus only on the first of these steps. There's an entire class, MATH 301 *Differential Equations*, that deals with the second step.

In the Phillips paper, the differential equations are labeled as Equations (1), (2), (3), and (4). **Your task is to understand the reasoning used to formulate these equations and to express that reasoning in your own terms.** The author's reasoning is given in the second paragraph of the paper.

The general idea behind this model is to think about some specific type of stuff in a container. (The container could be a real physical container or could be a conceptual container.) The amount of stuff in the container can change in time (by various processes). Let A represent some measurement of the amount of stuff. (For example, if the container is a tank and the stuff is water in the tank, we might measure the volume of the water or we might measure the mass of the water.) Since the amount of stuff in the container can change in time, we should think of A as a function of time t. The derivative dA/dt then represents the rate of change in the amount of stuff in the container. Each process contributes to the overall rate of change. So, if we identify independent processes, we can then write an equation in the form

$$\frac{dA}{dt} = \left(\begin{array}{c}\text{rate of change due to}\\ \text{first process}\end{array}\right) + \left(\begin{array}{c}\text{rate of change due to}\\ \text{second process}\end{array}\right) + \cdots + \left(\begin{array}{c}\text{rate of change due to}\\ \text{last process}\end{array}\right)$$

The rate of change due to each process might depend on A itself.

Equations (1)-(4) in the Phillips paper all have the general structure described in the previous paragraph. Your job is to understand and explain how these equations model the specific real-world phenomena in question. Here are some general questions that might guide your thinking:

- What is the container?
- What types of stuff in the container are being studied?
- For each type of stuff, what are the processes that change the amount of that stuff in the container?
- For each process, what is the corresponding term in the relevant differential equation?
- How does the form of each term relate to the process's contribution to the overall rate of change?