The Mean Value Theorem

1. (a) Set up a coordinate system and mark off a closed and bounded interval $[a, b]$ on the horizontal axis.

   (b) Draw the graph of a generic function f for the interval $[a, b]$. You do not need to start with a formula for $f(x)$. Just draw any curve that passes the vertical line test.

   (c) Draw the secant line through the points $(a, f(a))$ and $(b, f(b))$. Find an expression for the slope of this secant line.

   (d) Is there some input in the interval (a, b) for which the corresponding tangent line is parallel to the secant line through the points $(a, f(a))$ and $(b, f(b))$?

   (e) Repeat steps (a)–(d) with the goal of finding an interval $[a, b]$ and function f for which the answer to the question in (d) is the opposite of the answer you got on the first pass.

   (f) What condition or conditions on the function f will guarantee that the answer to the question in (d) is yes? Write your response in the form of an "if–then" statement.

2. A car traveling along a straight road goes from position 20 km to position 130 km in 2 hours. The average velocity of the car is thus $\frac{130\text{ km}-20\text{ km}}{2\text{ hr}} = 55$ km/hr. Is there some time in that two hour interval at which the (instantaneous) velocity of the car was exactly 55 km/hr?

3. Two swimmers in a 50 meter (one pool length) race start at the same time and finish in an exact tie. Must it be the case that there is at least one time during the race at which the swimmers had the same velocity?