**Exploring more limits**

In a previous exercise, you built evidence to support the following conjecture:

$$\textit{at } x = 0\textit{, the limit of } f(x) = \frac{\sin x}{x} \textit{ is } 1.$$

We arrived at this conjecture by choosing infinite lists of input values $x$ that "end" at 0 and finding that the corresponding lists of output values $f(x)$ all "end" at 1.

Use this same approach to make conjectures about each of the following. Each conjecture should be either a specific value or the statment "a limit does not exist".

1. the limit of $f(x) = \dfrac{\sqrt{x+4}-3}{x-5}$ at $x = 5$
2. the limit of $f(x) = \dfrac{x+3}{x^2-9}$ at $x = -3$
3. the limit of $f(x) = \dfrac{x+4}{x^2-9}$ at $x = -3$
4. the limit of $f(x) = \dfrac{\cos x}{x}$ at $x = 0$
5. the limit of $f(x) = \dfrac{1-\cos x}{x^2}$ at $x = 0$
6. the limit of $f(x) = \tan x$ at $x = \dfrac{\pi}{2}$
7. the limit of $f(x) = \sin(\frac{1}{x})$ at $x = 0$
8. the limit of $f(x) = x^2 + 1$ at $x = 2$
9. the limit of $f(x) = x$ at $x = 3$
10. the limit of $f(x) = 5$ at $x = 3$
11. the limit of $f(x) = \begin{cases} x^2 & \text{if } x < 4 \\ x+3 & \text{if } x > 4 \end{cases}$
12. the limit of $f(x) = \begin{cases} x^2 & \text{if } x < 4 \\ x+12 & \text{if } x > 4 \end{cases}$
13. the limit of $f(x) = \dfrac{3x}{x+10}$ at "$x = \infty$"
14. the limit of $f(x) = \dfrac{3x^2}{x+10}$ at "$x = \infty$"
15. the limit of $f(x) = \dfrac{3x}{x^2+10}$ at "$x = \infty$"
16. the limit of $f(x) = \cos(\frac{1}{x})$ at "$x = \infty$"