An example of Euler's method

We can use *linearization* to approximate a solution of a differential equation. For this example, we will look at the differential equation $R'(t) = -0.3R(t)$ and the initial condition $R(0) = 1000$.

Having the differential equation allows us to compute slopes of tangent lines. Since we are given $R(0)$, we can compute

$$R'(0) = -0.3R(0) = -0.3(1000) = -300.$$

With this, we can build the linearization based at $t = 0$ as

$$L(t; 0) = R(0) + R'(0)(t - 0) = 1000 - 300(t - 0).$$

To proceed, we must pick a time step Δt. Here, we use $\Delta t = 0.1$. We can now use the linearization based at 0 to approximate $R(0.1)$ as

$$R(0.1) \approx L(0.1; 0) = 1000 - 300(0.1 - 0) = 970.$$

Exercise 1: Build the linearization based at $t = 0.1$ using $R(0.1) = 970$.

Exercise 2: Use your linearization from Exercise 1 to approximate $R(0.2)$.

On the flip side, you'll find a table and chart showing the results from repeating linearization in this way for many steps. (The table ends at step 39 while the chart shows results for 100 steps.) Compare your result from Exercise 2 with the result in Step 2 of the table.

The process of approximating the solution to a differential equation by repeatedly linearizing is called *Euler's method*.

Step	t	R	R'
0	0.0	1000.00	-300.00
1	0.1	970.00	-291.00
2	0.2	940.90	-282.27
3	0.3	912.67	-273.80
4	0.4	885.29	-265.59
5	0.5	858.73	-257.62
6	0.6	832.97	-249.89
7	0.7	807.98	-242.39
8	0.8	783.74	-235.12
9	0.9	760.23	-228.07
10	1.0	737.42	-221.23
11	1.1	715.30	-214.59
12	1.2	693.84	-208.15
13	1.3	673.03	-201.91
14	1.4	652.84	-195.85
15	1.5	633.25	-189.98
16	1.6	614.25	-184.28
17	1.7	595.83	-178.75
18	1.8	577.95	-173.39
19	1.9	560.61	-168.18
20	2.0	543.79	-163.14
21	2.1	527.48	-158.24
22	2.2	511.66	-153.50
23	2.3	496.31	-148.89
24	2.4	481.42	-144.43
25	2.5	466.97	-140.09
26	2.6	452.97	-135.89
27	2.7	439.38	-131.81
28	2.8	426.20	-127.86
29	2.9	413.41	-124.02
30	3.0	401.01	-120.30
31	3.1	388.98	-116.69
32	3.2	377.31	-113.19
33	3.3	365.99	-109.80
34	3.4	355.01	-106.50
35	3.5	344.36	-103.31
36	3.6	334.03	-100.21
37	3.7	324.01	-97.20
38	3.8	314.29	-94.29
39	3.9	304.86	-91.46