Another new idea

1. For each of the following, sketch the graph of a function (defined for all inputs except possibly $x = 3$) that has the given properties.

   (a) $\lim\limits_{x \to 3} f(x) = 4$ $f(3)$ is undefined

   (b) $\lim\limits_{x \to 3} f(x) = 4$ $f(3) = 2$

   (c) $\lim\limits_{x \to 3} f(x) = 4$ $f(3) = 4$

   (d) $\lim\limits_{x \to 3^-} f(x) = 4$ $\lim\limits_{x \to 3^+} f(x) = 5$ $f(3)$ is undefined

   (e) $\lim\limits_{x \to 3^-} f(x) = 4$ $\lim\limits_{x \to 3^+} f(x) = 4$ $f(3)$ is undefined

   (f) $\lim\limits_{x \to 3^-} f(x)$ does not exist $\lim\limits_{x \to 3^+} f(x) = 4$ $f(3) = 4$

2. For which of the cases above can you draw the graph of the function through $x = 3$ without lifting your chalk (or pen or pencil)?

3. State a condition involving $f(3)$ and $\lim\limits_{x \to 3} f(x)$ that only the case you identify in Problem 2 satisfies among all of the cases in Problem 1.

4. For each of the cases in Problem 1, find a formula for a function that has the given properties.

5. Sketch a graph or find a formula for a function that has the following properties:

   $f(x)$ is defined for all x $\lim\limits_{x \to a} f(x)$ does not exist for each value of a