The derivative of a composition

Consider having a gas in a cylinder with an adjustable piston so that we can control or measure quantities such as volume V, pressure p, temperature T, and the number of gas atoms/molecules n. Each of these quantities has units as given in the table below.

Quantity	Units
volume V	liter (L)
pressure p	atmosphere (atm)
temperature T	Kelvin (K)
number n	mol

We'll assume that these quantities are related by the *ideal gas law*

$$pV = nRT$$

where R is a constant with value $R = 0.08206\,\frac{\mathrm{L\cdot atm}}{\mathrm{mol\cdot K}}$.

Let's consider a specific situation in which we keep the number of gas particles constant at $n = 1$ mol and the temperature constant at $T = 300$ K. We can then think of volume as a function of pressure. We can solve to get

$$V = nRT \cdot \frac{1}{p}$$

where nRT is a constant in the context we've set up.

Exercise 1: Compute the rate of change in volume with respect to pressure for $p = 1.09$ atm. Include units in your calculations.

Now, let's introduce time t as a new variable with t measured in minutes (min). Imagine that we control pressure over time. To be specific, suppose we control pressure so that

$$p = p_0 + at^2$$

with constants $p_0 = 1$ atm and $a = 0.01\,\frac{\text{atm}}{\text{min}^2}$.

Exercise 2(a): Compute the pressure for $t = 3$ min. Include units in your calculations.

Exercise 2(b): Compute the rate of change in pressure with respect to time for $t = 3$ min. Include units in your calculations.

Here's the main question: How do we combine

$$\left.\frac{dV}{dp}\right|_{p=1.09\text{ atm}} \qquad \text{and} \qquad \left.\frac{dp}{dt}\right|_{t=3\text{ min}}$$

to get the rate of change in volume *with respect to time* at $t = 3$ min? In other words, how do we get

$$\left.\frac{dV}{dt}\right|_{t=3\text{ min}} \qquad \text{from} \qquad \left.\frac{dV}{dp}\right|_{p=1.09\text{ atm}} \qquad \text{and} \qquad \left.\frac{dp}{dt}\right|_{t=3\text{ min}} \qquad ?$$

Exercise 3: Conjecture a relation that gives

$$\left.\frac{dV}{dt}\right|_{t=3\text{ min}} \qquad \text{in terms of} \qquad \left.\frac{dV}{dp}\right|_{p=1.09\text{ atm}} \qquad \text{and} \qquad \left.\frac{dp}{dt}\right|_{t=3\text{ min}}$$

Hint: Think about *units*.