**Part A Instructions:** Do the work for these problems on separate paper. Clearly box or circle the final answer for each computational problem. Do reasonable simplification. Each problem has a value of 5 points. You may not use the symbolic features of a calculator such as those on a TI-89 for this part of the exam.

1. State the definition of *derivative*.

2. State the definition of *definite integral*.

3. State the Second Fundamental Theorem of Calculus. Include the hypotheses and the conclusion.

4. Evaluate the limit $\lim\limits_{x \to 0} \dfrac{\sin(5x)}{3x}$.

5. Evaluate the limit $\lim\limits_{x \to 3} \dfrac{x^2 - 9}{x - 3}$.

6. Evaluate the limit $\lim\limits_{x \to \infty} \dfrac{(3x + 1)^2}{4x^2 + 7}$.

7. Determine if $f(x) = \begin{cases} 2x + 8 & \text{if } x \le 4 \\ x^2 & \text{if } x > 4 \end{cases}$ is continuous at $x = 4$.

8. Compute the derivative of $f(x) = x^4 - 7x^2 + 3$.

9. Compute the derivative of $g(z) = z \cos(z)$.

10. Compute the derivative of $f(t) = e^{t^2 + t}$.

11. Compute the derivative of $f(x) = \dfrac{x}{x^3 + 1}$.

12. Compute the derivative of $h(y) = \sin(\ln y)$.

13. Compute the derivative of $f(x) = A \sin(kx)$ where A and k are constants.

14. Compute the *second* derivative of $f(x) = \dfrac{5}{x^2} + \ln x$.

15. The plot below shows the graph of a function f. Below it are three other plots. Determine which of the three shows the graph of the derivative f'.

(a)
(b)
(c)
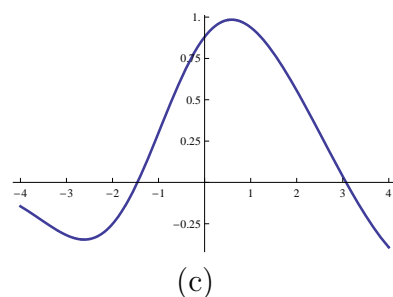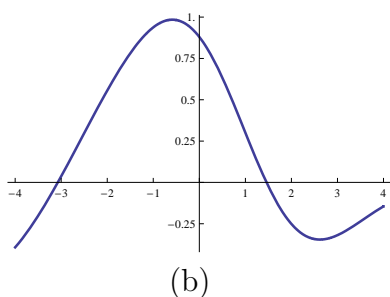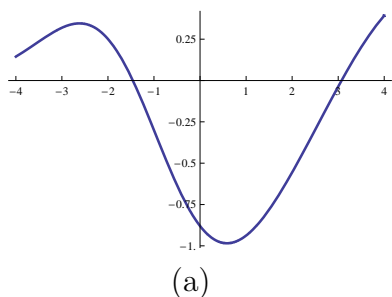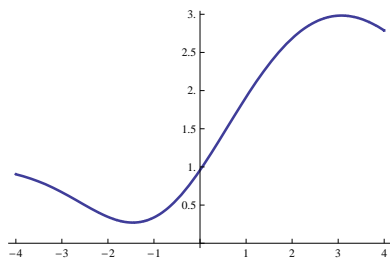
16. Find the slope of the curve $y = x + \sin x$ for $x = \pi$.

17. Find the equation of the line tangent to the graph of $f(x) = x^3 + 4x^2$ at $x = 3$.

18. The surface area S of a sphere of radius r is given by the formula $S = 4\pi r^2$. Find a formula for the rate of change in S with respect to r.

19. The critical values of the function $f(x) = 3x^4 - 16x^3$ are $x = 0$ and $x = 4$. Use calculus to classify the critical value $x = 4$ as a local minimum, a local maximum, or neither. Note: You do not need to classify the critical value $x = 0$.

20. Find an antiderivative of $g(t) = t^2 + e^t$.

21. Evaluate the indefinite integral $\displaystyle\int (\sqrt{x} + \cos x)\, dx$.

22. The acceleration of an object moving along one direction is given by $a(t) = 1 - t^2$. Find the velocity of the object given that $v(0) = 2$.

23. Evaluate the definite integral $\displaystyle\int_{-2}^{1} 4x^3 \, dx$.

24. Evaluate the definite integral $\displaystyle\int_{0}^{5} e^{-2x} \, dx$

25. A kindergartner is squeezing a long line of glue from a bottle at a rate given by $f(t) = t^2$ in units of inches per second. How long is the section of glue squeezed out between $t = 0$ and $t = 2$ seconds?

Part B Instructions: Do any three of the following six problems. Do the work for these problems on separate paper. On this page, circle each of the problem numbers for the three problems you are submitting. Each problem is worth 15 points.

I. Find the slope of the line tangent to the curve given by $x^2y - xy^3 = 12$ at the point $(x, y) = (4, 1)$.

II. Consider the function $f(x) = xe^{-ax}$ where a is a positive constant. Use calculus techniques to do the following.

   (a) Show that the function has a local maximum at $x = \dfrac{1}{a}$.

   (b) Show that the function has an inflection point at $x = \dfrac{2}{a}$.

III. Determine the derivative of the inverse cosine function given that we know the derivative of the cosine function. Find an expression that gives this derivative in terms of algebraic operations (as opposed to transcendental operations such as trigonometric, exponential, and logarthimic functions).

IV. You are watching a hot air balloon that is rising straight up at a constant rate of 30 feet per minute. You are on flat ground at a spot 200 feet from where the balloon was launched. How fast is the distance between you and the balloon changing at the time the balloon is 500 feet above the ground?

V. A window is to be made in the shape of a rectangle capped by a semicircle as shown in the figure. The perimeter is to be a total of 30 feet. Find the dimensions that maximize the area of the window. Note: Do not count the dashed line in the figure as part of the perimeter.
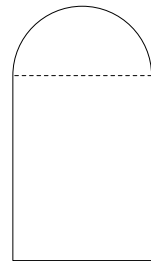
VI. Approximate the value of $\displaystyle\int_0^1 \dfrac{1}{1 + x^3}\,dx$ using 10 rectangles and right endpoints.