MATH 180B   Calculus and Analytic Geometry I   Fall 2008   Exam #5

Instructions: You can work on the problems in any order. Do your work on separate paper. You do not need to write answers on the question sheet.

   This exam is a tool to help me (and you) assess how well you are learning the course material. As such, you should report enough written detail for me to understand how you are thinking about each problem.   (100 points total)

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1. State a precise definition, equivalent to that given in the text, of L *is the limit of f at* $x = a$. For full credit, give a self-contained definition rather than one that uses words which need to be defined themselves.   (8 points)

2. Use the precise definition of limit to prove the limit statement $\lim_{x \to 4}(5x - 3) = 17$.   (7 points)

3. For each of the following, find all antiderivatives of the given function.   (10 points each)

   (a) $f(x) = x^3 + 5x^2 + \dfrac{4}{x}$ (b) $g(z) = e^{6z} - \cos(2z)$

4. The velocity of an object moving along one direction is given by $v(t) = \sin(t) - t$. Find the position function $s(t)$ for this object given that $s(0) = 4$.   (10 points)

5. Water is pumped into a tank at a flow rate given by $f(t) = e^{-t^2}$ for $t = 0$ to $t = 2$.

   (a) Express the volume of water that accumulates in the tank during this interval as a definite integral. (You just need to set up the relevant definite integral. You do not need to evaluate this definite integral.)   (4 points)

   (b) Estimate the volume of water that accumulates in the tank during this interval using an upper sum with 5 subintervals.   (9 points)

6. Use the definition of definite integral to evaluate $\displaystyle\int_0^2 x^3\,dx$. Hint: $\displaystyle\sum_{k=1}^{n} k^3 = \dfrac{n^2(n+1)^2}{4}$.

                                                                                                            (8 points)

7. State the Second Fundamental Theorem of Calculus.   (10 points)

8. Evaluate each of the following definite integrals.   (12 points each)

   (a) $\displaystyle\int_1^3 (x^2 - 2x)\,dx$ (b) $\displaystyle\int_0^{\pi} (t + \cos t)\,dt$