

Instructions: You can work on the problems in any order. Do your work on separate paper. You do not need to write answers on the question sheet.

This exam is a tool to help me (and you) assess how well you are learning the course material. As such, you should report enough written detail for me to understand how you are thinking about each problem. (100 points total)

1. State the Mean Value Theorem. (8 points)

2. You have a friend who complains about everything. Your friend even complains about having unsteady and low water flow in a garden hose at home. In fact, your friend claims there is never a flow rate of more than 1.5 gallons per minute. One hot summer day, you are watching your friend fill up a wading pool with the garden hose. The flow rate from the hose fluctuates a lot. You notice that your friend is able to fill up the 200 gallon pool in 120 minutes. Make an argument that your friend is wrong about the flow rate never being more than 1.5 gallons per minute. Include a relevant plot to illustrate your point. (8 points)

3. Evaluate each of the following limits.

Note: An informal argument will earn you some credit. For full credit, give a formal argument. (7 points each)

(a) $\lim_{x \rightarrow 0} \frac{x \sin x}{\cos x - 1}$

(b) $\lim_{x \rightarrow 1} \frac{\ln(x)}{x^{10} - 1}$

(c) $\lim_{x \rightarrow 0} (1 + 5x)^{1/x}$

4. The function $f(x) = x^3 - 6x^2$ has a critical point at $x = 4$. Use the second derivative to classify this critical point as a local minimum or a local maximum. (10 points)

5. Consider the function $f(x) = x^2 e^{-x}$. Use calculus techniques for each of the following. Show enough detail for a reader to understand how you reach your conclusions.

(a) Give the domain of this function. (3 points)

(b) Find all intervals of x for which the function is positive and all intervals of x for which the function is negative. (6 points)

(c) Find all intervals of x for which the function is increasing and all intervals of x for which the function is decreasing. (6 points)

(d) Find all intervals of x for which the function is concave up and all intervals of x for which the function is concave down. (6 points)

(e) Analyze any limits that are relevant in understanding the graph of f . (4 points)

(f) Sketch a graph of f and label any essential features (such as zeros, local extrema, and inflection points). Give x and y coordinates for each essential feature. (8 points)

6. You have a piece of wire that is 12 feet long. For an art project, you will cut the wire into two pieces. You will then bend one piece into a square and the other piece into a circle. Find where to cut the wire to get the smallest possible total area enclosed by square and circle.

Note: You can earn some credit by outlining a reasonable strategy even if you are unable to carry out the details of the strategy. (20 points)