Name ______________________________

MATH 180B Calculus and Analytic Geometry I Fall 2008 Exam #3

Instructions: You can work on the problems in any order. Do your work on separate paper. You do not need to write answers on the question sheet.

This exam is a tool to help me (and you) assess how well you are learning the course material. As such, you should report enough written detail for me to understand how you are thinking about each problem. (100 points total)

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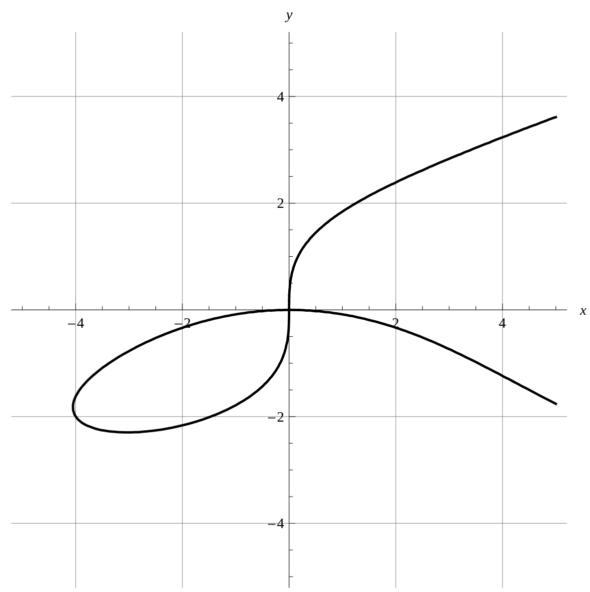
1. The plot below shows the graph of $x^3 - 2y^4 + 12xy = 0$.



   (a) From the plot, it appears that the point $(-4, -2)$ is on the graph. Use the equation to confirm that this is true. (2 points)

   (b) Compute the derivative $\dfrac{dy}{dx}$. (8 points)

   (c) Compute the slope of the tangent line to the graph at the point $(-4, -2)$. (2 points)

2. For each of the following, compute the derivative of the given function. (5 points each)

   (a) $f(x) = \ln(x^2 + 3)$

   (b) $y = e^x \ln x$

   (c) $g(x) = x \cos^{-1} x$

   (d) $p = \sin^{-1}(\theta^2)$

3. (a) Show that the derivative of $y = \tan^{-1} x$ is $\dfrac{dy}{dx} = \cos^2(\tan^{-1} x)$. (6 points)

   (b) Show that $\cos^2(\tan^{-1} x) = \dfrac{1}{1 + x^2}$. (3 points)

   (c) Put (a) and (b) together to conclude that the derivative of $y = \tan^{-1} x$ is $\dfrac{dy}{dx} = \dfrac{1}{1 + x^2}$. (1 point)

4. A railroad car is being pulled by a cable with no slack. The cable runs from the railroad car to a winch on a post at the end of the track. The vertical distance from the attachment point to the winch is 30 feet. A motor on the winch winds the cable in at a rate of 10 feet per minute. How fast is the railroad car moving along the track when it is 80 feet away from the end of the track? (16 points)

5. (a) Find the linearization for $f(x) = \ln x$ at $x = 1$. (8 points)

   (b) Use the linearization you found in (a) to approximate $\ln(0.92)$. (2 points)

6. The kinetic energy K of an object with mass m is related to velocity v by $K = \frac{1}{2}mv^2$. In an experiment, you might measure the velocity with some measurement error dv and thus a percentage measurement error dv/v. Use differentials to find a relationship between the percentage error in the computed value of K and the percentage error in the measured value of v. Assume m is constant. (8 points)

7. Determine the global minimum and the global maximum for the function $f(x) = \dfrac{x+1}{x^2+2x+2}$ on the interval $[-5, 5]$. For full credit, provide enough detail to make clear that you can do this without relying on graphing technology. (12 points)

8. Consider the function $f(x) = x + \dfrac{a}{x}$ where $a > 0$ is a constant.

   (a) Show that $x = \sqrt{a}$ is a critical value for f. (6 points)

   (b) Determine if f has a local minimum, a local maximum, or neither for $x = \sqrt{a}$. (6 points)