

Extreme value exercises

1. In Section 1.1 #62, we found a function that gives the cost C (in dollars) for a cable as a function of a certain distance x (in feet) to be

$$C(x) = 180\sqrt{x^2 + 800^2} + 100(10560 - x).$$

The relevant domain is $[0, 10560]$. Our goal is to find the distance x that gives the minimum cost.

- Use your calculator to estimate the minimizing distance and the minimum cost.
- Use calculus techniques to find the minimizing distance and the minimum cost.
- Compare the methods from (a) and (b). What are the relative advantages and disadvantages?
- In our expression for $C(x)$ above, we do not keep track of units. To do so, we should write

$$C(x) = (180 \text{ \$/ft})\sqrt{x^2 + (800 \text{ ft})^2} + (100 \text{ \$/ft})(10560 \text{ ft} - x).$$

We can write this in a simpler fashion if we introduce some parameters to represent the various constants. We'll use

$$a = 180 \text{ \$/ft}, \quad b = 100 \text{ \$/ft} \quad r = 800 \text{ ft}, \quad \text{and} \quad s = 10560 \text{ ft}.$$

With these, we have

$$C(x) = a\sqrt{x^2 + r^2} + b(s - x).$$

Find the minimizing distance using this expression.

- Find the global minimum and global maximum for $f(x) = xe^{-kx}$ on $[0, 10/k]$ where k is a positive constant.
- Find the global minimum and global maximum for $f(x) = x + \frac{B}{x}$ on $[0, 4\sqrt{B}]$ where B is a positive constant.