Curve sketching example

Consider the function

$$U(r) = \frac{A}{r^4} - \frac{B}{r^2}$$

where A and B are parameters with $A > 0$ and $B > 0$. Functions something like this are used in physics and chemistry as models of the *potential energy of interaction* between two molecules where r is the distance between the molecules. The relevant domain in this context is thus $(0, \infty)$.

1. Analyze the "endpoint" values for this function.
2. Determine the values of r for which U is positive and the values of r for which U is negative. As part of this, identify the location of all zeros for this function.
3. Determine the values of r for which U is increasing and the values of r for which U is decreasing. As part of this, identify the location of all local extremes for this function.
4. Determine the values of r for which U is concave up and the values of r for which U is concave down. As part of this, identify the location of all inflection points.
5. Use information from previous steps to sketch a plot that shows all of the essential features on the graph of U as a function of r.

You might find it useful to record some information in the table below.

r	$U(r)$	feature
0		limit or value at "endpoint"
∞		limit or value at "endpoint"