Undoing differentiation

1. (a) Find a function $F(x)$ with derivative equal to $f(x) = x^2$.

   (b) Find a different function $F(x)$ with derivative equal to $f(x) = x^2$.

   (c) Find all functions $F(x)$ that have derivative equal to $f(x) = x^2$.

   (d) Among the functions $F(x)$ you have in (c), find the one function with $F(6) = 10$.

   (e) Among the functions $F(x)$ you have in (c), find the one function with $F(0) = 2$.

2. (a) Find a function $P(t)$ with derivative equal to $p(t) = e^{3t}$.

   (b) Find a different function $P(t)$ with derivative equal to $p(t) = e^{3t}$.

   (c) Find all functions $P(t)$ that have derivative equal to $p(t) = e^{3t}$.

   (d) Among the functions $P(t)$ you have in (c), find the one function with $P(0) = 10$.

3. For each of the following, find all functions $F(x)$ with derivative equal to the given function $f(x)$.

   (a) $f(x) = 5x^2$

   (b) $f(x) = 3 + x$

   (c) $f(x) = 5x^2 + 7x - 2$

   (d) $f(x) = \sin x$

   (e) $f(x) = \cos x$

   (f) $f(x) = 5x + 3\sin x$

   (g) $f(x) = \sec^2 x$

   (h) $f(x) = \dfrac{1}{1+x^2}$

4. An object in free fall near the earth's surface has a constant acceleration of $-g$ where $g = 9.8$ m/s^2. If $a(t)$ is the object's acceleration function, we have $a(t) = -g$.

   (a) Find all velocity functions $v(t)$ corresponding to the acceleration function $a(t) = -g$.

   (b) Among the velocity functions you have in (a), find the one velocity function with $v(0) = 5$ m/s.

   (c) Among the velocity functions you have in (a), find the one velocity function with $v(0) = v_0$ where v_0 is a constant.

   (d) Find all position functions $s(t)$ corresponding to the velocity function you found in (c).

   (e) Among the position functions you have in (d), find the one position function with $s(0) = s_0$ where s_0 is a constant.