Probability of at least two people sharing a birthday in a group of size n

Consider a group of n people and ask for the probability that at least two share a birthday. Let A be the event of at least two sharing a birthday so A^c is the event of no shared birthdays. We can determine $P(A^c)$ directly. Note that the total number of birthday combinations is

$$365 \times 365 \times 365 \times \cdots \times 365 = 365^n$$

To determine the number of outcomes with no shared birthday, consider looking at birthdays in some order. The first person observed can have any of 365 birthdays. To avoid a shared birthday with the first, the second person can have any of 364 birthdays. The third person is limited to 363 birthdays to avoid conflict with the previous two. This pattern continues to give us the total number of outcomes with no shared birthday as

$$365 \times 364 \times 363 \times \cdots \times (365 - n + 1).$$

We then have

$$P(A^c) = \frac{365 \times 364 \times 363 \times \cdots \times (365 - n + 1)}{365^n}.$$

Values for $P(A^c)$ and $P(A) = 1 - P(A^c)$ are given in the table below for various values of n.

n	$P(A^c)$	$P(A)$
4	0.9836	0.0164
8	0.9257	0.0743
12	0.8330	0.1670
16	0.7164	0.2836
20	0.5886	0.4114
24	0.4617	0.5383
28	0.3455	0.6545
32	0.2467	0.7533
36	0.1678	0.8382
40	0.1088	0.8912
44	0.0671	0.9329
48	0.0394	0.9606
52	0.0220	0.9780
56	0.0117	0.9883
60	0.0059	0.9941