Name ____________________

MATH 310 | Numerical Analysis | Spring 2007 | Exam #3

Instructions:

Do your own work. You may consult your class notes and the course text. Do not consult other sources. Do not discuss generalities or specifics of the exam with anyone except me.

Turn in a complete and concise write up of your work. Show enough detail so that a peer could follow your work (both computations and reasoning). If you use computing technology, include relevant code, input, and output.

Each problem has a maximum value of 25 points.

The exam is due Friday, April 13.

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1. Approximate $\int_0^\infty \frac{1}{1+x^2+x^4}\,dx$ with a tolerance of 10^{-6} as your goal.

2. (a) Write a Mathematica implementation of the Runge-Kutta-Fehlberg method (as given in Algorithm 5.3 of the text). Organize your code so that it begins with all of the user-supplied input.

   (b) Use your implementation to approximate the solution of the initial-value problem

   $$\frac{dy}{dt} = 10y(1-y) \qquad\qquad y(0) = 0.01$$

   for $0 \le t \le 2$ with a tolerance of 10^{-8} as your goal. Use a minimum step size of 0.001 and a maximum step size of 0.1. Give a table of values and a plotof the approximate solution.

   (c) For your approximate solution from (b), make a plot of the step size vs. t. Explain the shape of this plot in reference to the initial-value problem or the approximate solution. In other words, explain why steps are smaller for some values of t than for other values of t.

3. Derive the Adams-Bashforth Five-Step Method and the local truncation error for this method (using the text's definition of local truncation error).

4. (a) Write a Mathematica implementation of the Adams Fourth-Order Predictor-Corrector method (given in Algorithm 5.4 of the text) with a variation that allows for iterating the corrector step m times with m supplied as a user input.

   (b) Use your implementation to approximate the solution of the initial-value problem

   $$\frac{dy}{dt} = 10y(1-y) \qquad\qquad y(0) = 0.01$$

   with a step size 0.1 and one iteration of the corrector method for $0 \le t \le 2$. Give a table of values and a plot of the approximate solution.

   (c) Repeat (b) with two iterations of the corrector step.

   (d) Repeat (b) with four iterations of the corrector step.

   (e) Esimate the number of iterations needed to get an approximation with a tolerance of 10^{-6} as your goal. (Do not use knowledge of the exact solution to justify this estimate.)