Problems from Section 2.1

1. Use the Bisection method to find p_3 for $f(x) = \sqrt{x} = \cos x$ on $[0, 1]$.
   Note: The text denotes the value of the nth approximation by p_n.

5. Use the Bisection method to find solutions accurate to with 10^{-5} for the following problems.

   (a) $x - 2^{-x} = 0 \quad \text{for} \quad 0 \leq x \leq 1$

7. (a) Sketch the graphs of $y = x$ and $y = 2 \sin x$.

   (b) Use the Bisection method to find an approximation to within 10^{-5} to the first positive value of x with $x = 2 \sin x$.

11. Let $f(x) = (x + 2)(x + 1)x(x - 1)^3(x - 2)$. To which zero of f does the Bisection method converge when applied on the following intervals?

    (a) $[-3, 2.5]$ (b) $[-2.5, 3]$ (c) $[-1.75, 1.5]$ (d) $[-1.5, 1.75]$

    Note: For each, try to determine the relevant zero with a minimal amount of computation. That is, try to avoid a "brute force" approach such as iterating the bisection method 1000 times and then checking which zero the resulting approximation is near.

18. The function $f(x) = \sin(\pi x)$ has zeros at every integer. Show that when $-1 < x < 0$ and $2 < b < 3$, the Bisection method converges to

    (a) 0, if $a + b < 2$ (b) 2, if $a + b > 2$ (c) 1, if $a + b = 2$

Programming Problem Modify the implementation of the bisection method from class to include a check that the original interval is valid.