Name ____________________

MATH 280A Multivariate Calculus Spring 2007 Project #2

Instructions

You should submit a carefully written report addressing the problems given below. You are encouraged to discuss ideas with others for this project. If you do work with others, you must still write your report independently.

Use the writing conventions given in *Some notes on writing in mathematics*. You should include enough detail so that a reader can follow your reasoning and reconstruct your work. You should not show every algebraic or arithmetic step. All graphs should be done carefully on graph paper or using appropriate technology.

For this project, submit a report on either one of the following.

The project report is due on Monday, March 26.

Complements, substitutes, and nrelated goods

Economists who study *consumer choice theory* use *consumer utility functions*. In class, we have used a specific consumer utility function as an example. Specifically, we consider a bundle of goods consisting of pizza, coffee, and textbooks. With p representing the amount of pizza, c representing the amount of coffee, and t representing the amount of textbooks in a bundle, the utility function we have used is

$$U(p, c, t) = p^{1/3}c^{1/2}t^{1/6}.$$

More generally, we could look at a bundle of n goods with amounts of each represented by x_1, x_2, ..., x_n. The utility for this bundle is denoted $U(x_1, x_2, \ldots, x_n)$. The formula we use for $U(x_1, x_2, \ldots, x_n)$ will depend on the nature of the goods in the bundle and the assumptions we make about how consumers operate.

For each pair of goods in a bundle, we can ask if the two goods are *complements*, *substitutes*, or *unrelated* with respect to a utility function. Your task is to find a definition of these concepts that is phrased in terms of *partial derivatives* of the utility function and to explain all of this to someone who has not previously heard of utility functions. Your explanation should include

- an intuitive idea of what *complements*, *substitutes*, and *unrelated* mean
- a mathematical defintition of *complements*, *substitutes*, and *unrelated*
- an explanation of how the mathematical definition relates to or captures the intuitive meaning

You should assume that your readers are in a multivariate calculus course but are not familiar with the idea of utility functions.

Spatial and time dependence in population models

Ecologists often model the way in which a population changes from place to place and time to time. For a species in which each individual has a fixed location (such a plant species), we can consider measuring the *population density* n as number of individuals per square meter. The density can vary with position and time. If we use cartesian coordinates (x, y) to measure position and let t denote time, the population density can be denoted $n(t, x, y)$. For simplicity, ecologists sometimes consider only one spatial variable in which case the population density is denoted $n(t, x)$.

For a specific species, the way in which n depends on t, x, and y is connected to various biological processes. It is often easiest to understand how each process contributes to the rate of change in n with respect to t. So, ecologists look at equations of the form

$$\frac{\partial n}{\partial t} = \begin{pmatrix} \text{contribution} \\ \text{from Process 1} \end{pmatrix} + \begin{pmatrix} \text{contribution} \\ \text{from Process 2} \end{pmatrix} + \ldots + \begin{pmatrix} \text{contribution} \\ \text{from Process n} \end{pmatrix}.$$

Your task is to find a specific example of an equation having this form and to explain it to someone who has not seen this type of population model. At least one of the terms on the right side of the equation should involve partial derivatives with respect to the spatial variables. Your explanation should include

- an intuitive description of each relevant biological process
- a mathematical expression for how each process contributes to the rate of change with respect to time
- an explanation of how each mathematical expression relates to or captures the relevant intuitive idea

You should assume that your readers are in a multivariate calculus course but are not familiar with spatially-dependent population models.