MATH 280   Multivariate Calculus   Fall 2006   Final Exam

Instructions: You can work on the problems in any order. Please use just one side of each page and clearly number the problems. You do not need to write answers on the question sheet.

  This exam is a tool to help me (and you) assess how well you are learning the course material. As such, you should report enough written detail for me to understand how you are thinking about each problem.

(100 points total)

1. (a) Give a *component* proof for the identity $\vec{u} \cdot (\vec{u} \times \vec{v}) = 0$.   (4 points)

   (b) Give a *geometric* argument for the identity $\vec{u} \cdot (\vec{u} \times \vec{v}) = 0$.   (4 points)

2. Consider the two curves given by the vector-output functions $\vec{R}(t) = t\,\hat{\imath} + t^2\,\hat{\jmath} + t^3\,\hat{k}$ and $\vec{P}(s) = (s+3)\,\hat{\imath} + (2s^2+2)\,\hat{\jmath} + (9-s^4)\,\hat{k}$.

   (a) Confirm that each curve contains the point $(2, 4, 8)$.   (3 points)

   (b) Find the angle between the curves at the point of intersection $(2, 4, 8)$.
       Hint: Think tangent vectors.   (7 points)

3. Find the equation of the tangent plane for $f(x, y) = x^3 y$ for $(2, 5)$.   (10 points)

4. Show that $(3, -1)$ is a critical input for the function $f(x, y) = x^2 y^3 - 6xy - 9y$ and classify this input as a local minimizer, a local maximizer, or neither.   (10 points)

5. Motion of air in the atmosphere (i.e., wind) is related to differences in air pressure from one place to another. In a simple-minded way of thinking about this, air is pushed in the direction that air pressure decreases most rapidly at each point. Find the direction air is pushed at the point $(2, 4, 1)$ if the air pressure is given by $p(x, y, z) = 4xy + 3yz$ (in unspecified units). Give the result as a unit vector.   (10 points)

There is more on the flip side.

6. Compute the value of the double integral $\iint\limits_{D} x^2 y \, dA$ where D is the region in the xy-plane bounded below by $y = x^3$ and above by $y = \sqrt{32x}$. (10 points)

7. Compute the formula for the volume of a sphere of radius R using ideas from this course. (10 points)

8. Compute the formula for the surface area of a sphere of radius R using ideas from this course. (10 points)

9. Consider the vector field $\vec{F}(x, y) = yz \, \hat{\imath} + xz \, \hat{\jmath} + xy \, \hat{k}$.

   (a) Compute the divergence of $\vec{F}$. (4 points)

   (b) Compute the curl of $\vec{F}$. (4 points)

   (c) Imagine a drop of dye in a fluid moving with velocities given by $\vec{F}$. Describe how the volume of the drop changes and how the drop rotates as it moves with the fluid. (2 points)

10. Consider the line integral $\int_{C} \vec{F} \cdot d\vec{R}$ where $\vec{F}(x, y, z) = z \, \hat{\imath} + z \, \hat{\jmath} + (x + y) \, \hat{k}$ and C is the straight line from $(2, 1, 3)$ to $(5, 0, 1)$.

   (a) Compute $\int_{C} \vec{F} \cdot d\vec{R}$ by parametrizing the curve. (6 points)

   (b) Compute $\int_{C} \vec{F} \cdot d\vec{R}$ using the Fundamental Theorem for Line Integrals. (6 points)