MATH 280   Multivariate Calculus   Fall 2006   Exam #4

Instructions: You can work on the problems in any order. Please use just one side of each page and clearly number the problems. You do not need to write answers on the question sheet.

    This exam is a tool to help me (and you) assess how well you are learning the course material. As such, you should report enough written detail for me to understand how you are thinking about each problem.

(80 points total)

1. Set up an interated integral (or integrals) equal to the triple integral $\iiint\limits_D z\, dV$ where

    D is the solid region bounded between the cone $z^2 = x^2 + y^2$ and the sphere of radius 5 centered at the origin with $z \geq 0$. Express your result entirely in terms of a single coordinate system (of your choice). You do not need to evaluate the integral or integrals.

    (12 points)

2. Set up a definite integral (or integrals) in one variable equal to the line integral $\int_C \vec{F} \cdot d\vec{R}$ where

    $$\vec{F} = 2y\,\hat{\imath} + z\,\hat{\jmath} + 5\,\hat{k}$$

    and C is the helix from $(0,0,0)$ to $(0,0,4)$ parametrized by

    $$\vec{R}(t) = (\cos t)\,\hat{\imath} + (\sin t)\,\hat{\jmath} + \frac{t}{\pi}\,\hat{k}.$$

    You do not need to set up or evaluate the integrals.

    (12 points)

3. Charge is distributed on the lateral surface of a right circular cylinder of radius r_0 and height h_0. The area charge density is proportional to the distance from one end of the cylinder. Compute the total charge on the cylinder.

    (12 points)

4. The accompanying figure shows a vector field $\vec{F}$, an oriented curve C (from P to Q), and points U and V.

    (a) Determine if the value of $\int_C \vec{F} \cdot d\vec{R}$ is positive or negative. Explain how you arrive at your conclusion.

    (3 points)

    (b) Determine if the divergence of $\vec{F}$ at U is positive or negative. Explain how you arrive at your conclusion.

    (3 points)

    (c) Determine if the $\hat{k}$-component of the curl of $\vec{F}$ at V is positive or negative. Explain how you arrive at your conclusion.

    (3 points)

5. Consider the vector field $\vec{F} = xz\,\hat{\imath} + xy^2z\,\hat{\jmath} - x^2z^3\,\hat{k}$.

   (a) Compute the divergence of $\vec{F}$ for the point $(2, -1, 1)$. (7 points)

   (b) Consider $\vec{F}$ as the velocity field for fluid flow. Imagine a small drop of dye placed at the point $(2, -1, 1)$. Describe how the volume of the drop will change (instantaneously) as the particles move with the flow. (3 points)

   (c) Compute the curl of $\vec{F}$ for the point $(2, -1, 1)$. (7 points)

   (d) Consider $\vec{F}$ as the velocity field for fluid flow. Imagine a small paddlewheel placed at the point $(2, -1, 1)$. Compare the rotation of the paddlewheel when its axis is in the $\hat{\imath}$-direction with the rotation of the paddlewheel when its axis is in the $\hat{k}$-direction. (3 points)

   (e) Is $\vec{F}$ conservative? Explain how you arrive at your conclusion. (3 points)

6. Evaluate $\displaystyle\int_C \vec{F} \cdot d\vec{R}$ where $\vec{F} = \dfrac{1}{y^2}\,\hat{\imath} + \left(5 - \dfrac{2x}{y^3}\right)\hat{\jmath}$ and C is any curve that starts at $(2, 3)$ and ends at $(-1, 1)$. (12 points)