Name ____________________

MATH 280 Multivariate Calculus Fall 2006 Exam #3

Instructions: You can work on the problems in any order. Please use just one side of each page and clearly number the problems. You do not need to write answers on the question sheet.

This exam is a tool to help me (and you) assess how well you are learning the course material. As such, you should report enough written detail for me to understand how you are thinking about each problem. (100 points total)

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1. The accompanying figure gives level curves for a function that gives the temperature at points in the rectangle $[-10, 10] \times [-10, 10]$ in the xy-plane. Assume temperature is measured in °C and distances are measured in meters.
   (a) On the figure, draw the gradient vector for each of the points $(4, 2)$ and $(-8, -4)$. Choose your own scale for the magnitude of gradient vectors. Use the same scale for both vectors you draw. (6 points)
   (b) Estimate all the points at which the gradient vector is zero. (6 points)

2. The temperature on a tabletop is given by $T(x, y) = k\,xy$ where $k = 15°\text{C/m}^2$ and (x, y) are cartesian coordinates measured in meters from one corner of the table. A bug passes through the point $(0.5, 0.2)$ m moving with velocity $0.03\hat{\imath} - 0.05\hat{\jmath}$ m/s. Find the rate of change in temperature with respect to time for this bug as it passes through $(0.5, 0.2)$ m. (8 points)

3. Consider the function $f(x, y) = x^2y - 10y$.
   (a) Find the greatest rate of change in f at the point $(4, 1)$. (6 points)
   (b) Find the rate of change in f at the point $(4, 1)$ in the direction of the point $(1, -5)$. (6 points)

4. Consider the function $f(x, y) = \dfrac{48}{xy} - 3x^2 - 12y^2$.
   (a) Show that $(-2, 1)$ is a critical point for f. (6 points)
   (b) Use the second derivative test to determine if $(-2, 1)$ is a local minimizer, a local maximizer, or neither. (6 points)

5. Find three positive numbers so that the first plus twice the second plus three times the third equals 26 and the product of the three is maximized. (12 points)

There is more on the flip side.

6. Set up, but do not evaluate, an interated integral (or integrals) equal to the double integral $\iint_D xy^2\,dA$ where D is the region in the first quadrant of the xy-plane bounded by $y = 0$, $y = x^2$, and $y = 6 - x$. Make sure you are looking at the correct region. Each edge of the region should be on one of the given curves. (12 points)

7. Charge is distributed in the planar region bounded by the curves $y = x^3$, and $y = \sqrt{x}$. The surface charge density is proportional to the distance from the x-axis (not the distance from the origin). Compute the total charge. (12 points)

8. Charge is distributed in the planar region formed by the "petal" of the curve $r = 2\sin(3\theta)$ that lies in the first quadrant. The surface charge density is proportional to the distance from the origin. Set up an iterated integral (or integrals) to compute the total charge. Express your result entirely in terms of a single coordinate system (of your choice). You do not need to evaluate the integral or integrals. (12 points)

9. (a) Explain the distinction between a *double integral* and an *iterated integral*. (4 points)

   (b) What does Fubini's theorem tells us about the relationship between the double integral $\iint_R f\,dA$ and the iterated integral $\int_a^b \int_{g(x)}^{h(x)} f(x, y)\,dydx$ where R is the region described by $a \le x \le b$ and $g(x) \le y \le h(x)$? (4 points)