Name _____

MATH 280 Multivariate Calculus Fall 2006 Exam #2

**Instructions:** You can work on the problems in any order. Please use just one side of each page and clearly number the problems. You do not need to write answers on the question sheet.

This exam is a tool to help me (and you) assess how well you are learning the course material. As such, you should report enough written detail for me to understand how you are thinking about each problem. (100 points total)

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1. Sketch and/or describe the surface given by $\frac{1}{3}x^2 - y^2 + \frac{1}{16}z^2 = 1$. (10 points)

2. Parametrize the curve formed by the intersection of the cylinder $x^2 + z^2 = 25$ and the plane $4x + y - 2z = 3$. Include a range for the parameter you use. (10 points)

3. The position of a fly buzzing around the room is given by

$$\vec{R}(t) = (\tfrac{1}{3}t^3 - 9t + 2)\,\hat{\imath} + (\tfrac{1}{2}t^2 - 3t - 4)\,\hat{\jmath} + (5t + t^2)\,\hat{k}$$

(in inches) where $\hat{\imath}$ points due east, $\hat{\jmath}$ points due south, and $\hat{k}$ points up.

   (a) Where is the fly at $t = 2$? (4 points)

   (b) Compute the velocity function for the fly. (4 points)

   (c) Find a point at which the fly is heading straight up. (4 points)

4. Captain Jack Sparrow fires a cannon from the edge of the deck of the *Black Pearl*. The cannon is inclined at an angle of 30° above horizontal and the cannon ball leaves the cannon at a speed of 46 meters per second.

   (a) Find the vector-output function for the position of the cannon ball as a function of time (ignoring air resistance). Specify the origin of the coordinate system you use. (7 points)

   (b) The *Interceptor* is parallel to the *Black Pearl* in the direction the cannon is fired. There is a gap of 200 meters between the ships and the Interceptor is 10 meters wide. Does the cannon ball hit the deck of the *Interceptor*? You can assume the decks of the two ships are the same height above the water. You can use 9.8 $\text{m/s}^s$ for the acceleration due to gravity. (3 points)

5. Consider the function $f(x, y) = \dfrac{y^3}{x}$.

   (a) Give the domain and range of this function. (5 points)

   (b) Sketch level curves for $z = -2, -1, 0, 1, 2$. (5 points)

*There is more on the flip side.*

6. Consider the limit $\lim_{(x,y)\to(0,0)} \dfrac{x^2y}{x^5+y^3}$.

   (a) Analyze the path limit along $y = 0$. (4 points)

   (b) Analyze the path limit along $y = x$. (4 points)

   (c) What, if anything, can you conclude from these two results about $\lim_{(x,y)\to(0,0)} \dfrac{x^2y}{x^5+y^3}$? Explain how you reach a conclusion. (4 points)

7. State the definition of the partial derivative $\dfrac{\partial H}{\partial q}$ for the function $H(p, q)$. (Give a definition, not an interpretation such as rate of change or slope. Note that the definition involves a difference quotient.) (6 points)

8. Compute all of the first and second partial derivatives for the function $f(x, y) = xy^2 + e^{xy}$. (12 points)

9. Consider a function $f : \mathbb{R}^3 \to \mathbb{R}$ for which we know the following information:

$$f(2, 4, -1) = 5, \quad \frac{\partial f}{\partial x}(2, 4, -1) = 2, \quad \frac{\partial f}{\partial y}(2, 4, -1) = -3, \quad \text{and} \quad \frac{\partial f}{\partial z}(2, 4, -1) = 6.$$

   Estimate $f(1.8, 4.1, -0.7)$. (8 points)

10. Do either one of the following two problems. Circle the number of the problem you are submitting. (10 points)

    (A) Find the equation of the tangent plane for $f(x, y) = x^3y^2$ for the point $(x_0, y_0) = (1, 2)$.

    (B) The gravitational force $F$ on a particle of mass $m$ at a distance $r$ from the center of the earth is given by the formula

$$F = k\frac{m}{r^2}$$

    where $k$ is a constant. Relate the percentage change in $F$ to percentage changes in $m$ and $r$.