Name ______________________________

MATH 280 Multivariate Calculus Fall 2006 Exam #1

Instructions: You can work on the problems in any order. Please use just one side of each page and clearly number the problems. You do not need to write answers on the question sheet.

This exam is a tool to help me (and you) assess how well you are learning the course material. As such, you should report enough written detail for me to understand how you are thinking about each problem. (80 points total)

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1. (a) Draw a figure illustrating two vectors $\vec{u}$ and $\vec{v}$ and the vector $3\vec{u} - 2\vec{v}$. (5 points)

   (b) Compute the components of $3\vec{u} - 2\vec{v}$ for $\vec{u} = 4\,\hat{\imath} - 5\,\hat{\jmath} + 10\,\hat{k}$ and $\vec{v} = -\,\hat{\imath} + 7\,\hat{\jmath} + 2\,\hat{k}$. (5 points)

2. For each of the following, give both a geometric definition and a component expression:

   (a) the dot product of $\vec{u}$ and $\vec{v}$ (5 points)

   (b) the cross product of $\vec{u}$ and $\vec{v}$ (5 points)

3. Find an equation for the plane that contains the point $Q(6, 1, 3)$ and the line given by the parametric equations $x = 3 + 4t$, $y = 7 - 2t$, and $z = 5t$. (12 points)

4. Find an equation (symmetric or parametric, your choice) for the line of intersection for the two planes given by $12x - 3y + z = 14$ and $x + y + z = -2$. (12 points)

5. Compute the volume of the parallelpiped defined by the vectors $7\,\hat{\imath} - 2\,\hat{\jmath} + \hat{k}$, $2\,\hat{\imath} + \hat{\jmath} - \hat{k}$, and $2\hat{\imath} - 3\,\hat{\jmath} + 2\hat{k}$. (8 points)

6. (a) To compute the distance d between a point P and a plane, we have been using the formula

$$d = \frac{|\overrightarrow{QP} \cdot \vec{N}|}{\|\vec{N}\|}$$

   where Q is any point on the plane and $\vec{N}$ is a normal vector for the plane. Give an argument to justify this formula. (6 points)

   (b) Compute the distance between the point $P(4, 3, -1)$ and the plane given by the equation $6x + 2y - 3z = 12$. (8 points)

There is more on the flip side.

7. Do either one of the following two problems. Circle the number of the problem you are submitting. (8 points)

   (A) Find the angle between the two planes $x - 2y - 5z = 3$ and $4x + 3y - 6z = 10$. Hint: It might help to draw a picture from the view of looking straight down the line along which two planes intersect.

   (B) In a cube, a body diagonal goes from one corner to the opposite corner through the body of the cube. A face diagonal goes from one corner to the opposite corner on the same face. Find the angle between a body diagonal and a face diagonal that meet at the same corner.

8. Do either one of the following two problems. Circle the number of the problem you are submitting. (6 points)

   (A) Prove the following: If $\vec{u}$, $\vec{v}$, and $\vec{w}$ are nonzero vectors, then $(\vec{u} \times \vec{v}) \times (\vec{u} \times \vec{w})$ is parallel to $\vec{u}$.

   (B) Prove the following: If $\vec{u}$ and $\vec{v}$ are nonzero vectors, then $\|\vec{v}\|\vec{u} + \|\vec{u}\|\vec{v}$ and $\|\vec{v}\|\vec{u} - \|\vec{u}\|\vec{v}$ are perpendicular.