Name ______________________________

MATH 352 Complex Analysis Spring 2006 Exam #2

Instructions: Use separate paper for your responses. Please use just one side of each sheet and clearly number the problems. You can work on the problems in any order. Each statement of an axiom, definition, or theorem should be equivalent to that in the text.

This exam is a tool to help me (and you) assess how well you are learning the course material. As such, you should report enough written detail for me to understand how you are thinking about each problem.

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1. For each of the following, state a definition equivalent to that used in the text or class. (5 points each)

   (a) the function f is analytic on the domain D

   (b) $\int_C f(z)\,dz$ where C is a contour

2. For each of the following, compute the value, giving your result in the form $a+ib$ where a and b are real. (5 points each)

   (a) $\mathrm{Log}(-1-i\sqrt{3})$ (b) $\cos(3+2i)$ (c) e^{e^i}

3. For each of the following, either prove the given statement is an identity or give a counterexample to show it is not an identity. (8 points each)

   (a) $e^{\log z} = z$ (b) $\log(e^z) = z$

4. Prove the identity $\cos^2 z + \sin^2 z = 1$ for all z in $\mathbb{C}$. (8 points)

5. Determine the largest domain for which each of the following functions is analytic. (7 points each)

   (a) $f(z) = \dfrac{z}{\sin(iz)}$ (b) $f(z) = \mathrm{Log}(\mathrm{Log}\, z)$

6. Suppose f is analytic at $2+3i$ with $f(2+3i) = 6-5i$ and $f'(2+3i) = 1-4i$. Make a plot of the z-plane showing the point $2+3i$ and a small (or infinitesimal) rectangle with lower left corner at $2+3i$ and sides parallel to the real and imaginary axes of lengths dx and dy, respectively. Make a plot of the w-plane showing the (approximate) image of this rectangle under the function f. Use the same scale for both planes. (7 points)

7. For each of the following, evaluate the given contour integral.

(a) $\displaystyle\int_{C_1} \frac{1}{z}\, dz$ where C_1 is the line segment from 1 to $e^{2\pi}$ (8 points)

(b) $\displaystyle\int_{C_2} \frac{1}{z}\, dz$ where C_2 is the exponential spiral from 1 to $e^{2\pi}$ given by the polar relation $r = e^{\theta}$ (8 points)

(c) $\displaystyle\int_{C} \frac{1}{z}\, dz$ where C is the closed contour starting at 1, going to $e^{2\pi}$ along the exponential spiral of (b), and returning to 1 along a line segment (6 points)

8. Evaluate $\displaystyle\int_C \cos z\, dz$ where C is the semicircle from i to $-i$ to the left of the imaginary axis. (8 points)