Name ______________________

MATH 352 | Complex Analysis | Spring 2006 | Exam #1

Instructions: Use separate paper for your responses. Please use just one side of each sheet and clearly number the problems. You can work on the problems in any order. Each statement of an axiom, definition, or theorem should be equivalent to that in the text.

This exam is a tool to help me (and you) assess how well you are learning the course material. As such, you should report enough written detail for me to understand how you are thinking about each problem.

---

1. Give as many different representations of $4 - 2i$ as you can. (6 points)

2. For each of the following, simplify the given number into the form $x + iy$ where x and y are real. (5 points each)

   (a) $\dfrac{7-2i}{3+4i}$ (b) $(-1+i)^8$ (c) $(7+2i)e^{i\pi/3}$

3. Find the 4th roots of $-\frac{5}{2} + \frac{5\sqrt{3}}{2}i$. Identify the principal root. You can express the roots in cartesian, polar, or exponential form. (10 points)

4. Let $S = \left\{ z \middle| z = re^{i\theta}, \frac{1}{2} \le r < 3, \pi \le \theta \le \frac{3\pi}{2} \right\}$.

   (a) Sketch S. (4 points)

   (b) Is S open, closed, or neither? Is S bounded or unbounded? Is S a domain? Is S a region? (8 points)

   (c) Sketch the image of S for the function $f(z) = z^3$. (8 points)

5. (a) State a precise definition of w_0 *is the limit of* $f(z)$ *at* z_0 (6 points)

   (b) Use the definition to prove the following statement: If $\lim_{z \to z_0} f(z) = w_0$ and a is any complex number, then $\lim_{z \to z_0} (af(z)) = aw_0$. (8 points)

6. State a precise definition of f *is differentiable at* z *with derivative* $f'(z)$. (6 points)

7. Consider the function $f(z) = \dfrac{1}{z}$ defined for $z \neq 0$.

   (a) Using the definition of derivative, show that $f(z)$ is differentiable for all $z \neq 0$ and compute $f'(z)$. (8 points)

   (b) Using the real and imaginary parts of f, show that $f(z)$ is differentiable for all $z \neq 0$ and compute $f'(z)$. (8 points)

8. (a) Prove the identity $\dfrac{1}{z} = \dfrac{\bar{z}}{|z|^2}$. (6 points)

   (b) Consider the function $f(z) = \dfrac{1}{z}$. Use the identity in (a) to show geometrically how f maps a typical point z_0 with $|z_0| < 1$, a typical point z_1 with $|z_1| = 1$, and a typical point z_2 with $|z_2| > 1$. (6 points)