Name ______________________________

MATH 352 Complex Analysis Spring 2003 Exam #3

**Instructions:** Do your own work. You may consult class notes, the course text, or other books. Give a reference if you use some source other than class notes or the course text. Turn in a complete and concise write up of your work. Show enough detail so that a peer could follow your work. If you are not confident in some result, you will receive more credit if you make a note of this and comment on where you might be going wrong or on alternate approaches you might try. The exam is due Thursday, April 10 at 8:30 am.

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1. Let $C$ be the unit circle centered at the origin oriented counterclockwise. (24 points)

   (a) Find the value of $\displaystyle\int_C \frac{\log z}{z}\,dz$ with the branch using $-\pi < \arg z \leq \pi$ for the logarithm.

   (b) Find the value of $\displaystyle\int_C \frac{\log z}{z}\,dz$ with the branch using $0 \leq \arg z < 2\pi$ for the logarithm.

2. Evaluate $\displaystyle\int_C \frac{11z^2 + 10z - 162}{z^3 - z^2 - 22z + 40}\,dz$ where $C$ is the circle of radius 3 centered at the origin oriented counterclockwise. Hint: Rewrite the integrand using *partial fractions*. Most calculus books explain the algebra of partial fractions. (22 points)

3. Let $C_R$ be the circle of radius $R$ centered at the origin. Find an upper bound on $\displaystyle\left|\int_{C_R} \frac{e^z}{z}\,dz\right|$ *without* evaluating the contour integral explicitly. (22 points)

4. Problem #8 on page 129. Come talk with me if you are not familiar with the binomial formula suggested as a hint in the problem. (18 points)

5. Prove the following: If $f$ is entire and $\mathrm{Im}(f(z)) \leq 0$ for all $z$, then $f$ is a constant function. (14 points)