Name ____________________

MATH 352 Complex Analysis Spring 2003 Exam #2

Instructions: Use separate paper for your responses. Please use just one side of each sheet and clearly number the problems. You can work on the problems in any order. Each statement of an axiom, definition, or theorem should be equivalent to that in the text.

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1. For each of the following, compute all values of the given expression (giving results in the form $a + bi$). Show enough detail so it is clear you can do this without the aid of a calculator. (4 points each)

   (a) $e^{\sqrt{3}-i}$

   (b) $\sin(\sqrt{3}-i)$

   (c) $\log(\sqrt{3}-i)$

   (d) $\sinh(\sqrt{3}-i)$

2. Let $f(z) = (x^2 + 2y) + i(x^2 + y^2)$ for $z = x + iy$.

   (a) At what points is f differentiable? (6 points)

   (b) At what points is f analytic? (6 points)

3. Consider the function $f(z) = e^{-z^2}$.

   (a) Compute $f'(i)$. (6 points)

   (b) Use the result for (a) to describe the geometric effect of the mapping f near the point $z = i$. (6 points)

4. (a) Find the real and imaginary parts of $f(z) = e^{e^z}$. (6 points)

   (b) Give an argument to show that $f(z) = e^{e^z}$ is entire.
   Hint: Don't even think about using the Cauchy-Riemann equations. (4 points)

5. Consider the function $u(x, y) = \dfrac{y}{x^2 + y^2}$ for $(x, y) \neq (0, 0)$.

   (a) Show that $u(x, y)$ is harmonic. (4 points)

   (b) Find a harmonic conjugate $v(x, y)$ for $u(x, y)$. (6 points)

   (c) Express the function $f(z) = u(x, y) + iv(x, y)$ in terms of z. (4 points)

6. Find all values of $(1 + i)^i$ and plot these values (or at least plot enough to make clear where these are). (12 points)

7. Prove the identity $\cos^2 z + \sin^2 z = 1$. (12 points)

8. (a) Let z, α, and β be complex numbers. Prove the following: If z^α, z^β, and $z^{(\alpha+\beta)}$ are all defined using the same branch of log, then $z^\alpha z^\beta = z^{(\alpha+\beta)}$. (6 points)

   (b) Let z_1, z_2, and α be complex numbers. Prove the following: Branches for $\log(z_1)$, $\log(z_2)$, and $\log(z_1 z_2)$ can be chosen so that $(z_1 z_2)^\alpha = z_1^\alpha z_2^\alpha$. (6 points)