Name ____________________

MATH 352 Complex Analysis Spring 2003 Exam #1

Instructions: Use separate paper for your responses. Please use just one side of each sheet and clearly number the problems. You can work on the problems in any order. Each statement of an axiom, definition, or theorem should be equivalent to that in the text.

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1. For each of the following, simplify the given number into the form $a + ib$. (4 points each)

   (a) $\dfrac{3-2i}{4+5i}$ (b) $(1-i)^{10}$ (c) $(7+2i)e^{i\pi/3}$

2. Compute the modulus of $\dfrac{(4+3i)^3}{(6-i)^2}$ (8 points)

3. For each of the following, either prove the identity or give a counterexample to show the identity is false. (5 points each)

   (a) $\operatorname{Re}(zw) = \operatorname{Re}(z)\operatorname{Re}(w)$ (b) $|zw| = |z||w|$

4. Find the 5th roots of $-1 + i\sqrt{3}$. Hint: $\tan\frac{\pi}{3} = \sqrt{3}$. (10 points)

5. Find the image in the w-plane of the region in the z-plane with $1 \le r \le 2$ and $0 \le \theta \le \frac{\pi}{2}$ under the cubing function $w = z^3$. Sketch both the region and its image. (10 points)

6. Prove the following statement: If $|z| < 3$, then $|z^3 - 4z^2 + 3| < 66$. (8 points)

7. (a) State the definition of $\lim_{z\to z_0} f(z) = w_0$. (6 points)

   (b) Give a direct proof of the limit $\lim_{z\to 0} \dfrac{5\bar{z}^2}{z} = 0$. (6 points)

8. Consider the function $f(z) = z + \dfrac{1}{z}$. Find $u(x,y)$ and $v(x,y)$ where $f(x+iy) = u(x,y) + iv(x,y)$. (10 points)

9. Prove that $f(z) = z + \dfrac{1}{z}$ is continuous for $z_0 \neq 0$. You can refer to limit theorems in doing this. (8 points)

10. (a) Prove the identity $\dfrac{1}{z} = \dfrac{\bar{z}}{|z|^2}$. (6 points)

    (b) Consider the function $f(z) = \dfrac{1}{z}$. Use the identity in (a) to show geometrically how f maps a point z_0 with $|z_0| < 1$, a point z_1 with $|z_1| = 1$, and a point z_2 with $|z_2| > 1$. (6 points)