Computing contour integrals

Evaluate each of the following contour integrals.

1. $\displaystyle\int_{C_1} z\,dz$ where C_1 is the quarter-circle from $(1,0)$ to $(0,1)$

2. $\displaystyle\int_{C_2} z\,dz$ where C_2 is the line segment from $(1,0)$ to $(0,1)$

3. $\displaystyle\int_{C_1} \bar{z}\,dz$ where C_1 is the quarter-circle from $(1,0)$ to $(0,1)$

4. $\displaystyle\int_{C_2} \bar{z}\,dz$ where C_2 is the line segment from $(1,0)$ to $(0,1)$

5. $\displaystyle\int_{C_3} \frac{1}{z}\,dz$ where C_3 is the unit circle oriented counter-clockwise

6. $\displaystyle\int_{C_4} \frac{1}{z}\,dz$ where C_4 is the square oriented counter-clockwise with vertices at $(1,1)$, $(-1,1)$, $(-1,1)$ and $(1,-1)$

Based on these examples, make general conjectures about contour integrals.

Comments on Problem 6:

- You can break the square into four line segments and deal with each segment separately.

- The safe way to deal with each segment is to pull the relevant integral apart into real and imaginary parts. Doing so, you will end up with integrals along the lines of
$$\int_{-1}^{1} \frac{t}{1+t^2}\,dt \qquad \text{and} \qquad \int_{-1}^{1} \frac{1}{1+t^2}\,dt.$$
The first of these can be easily evaluated by noting the symmetry of the integrand and the symmetry of the interval. For the second, note that
$$\frac{d}{dt}[\tan^{-1} t] = \frac{1}{1+t^2}.$$

- An alternative is to deal with integrals along the lines of
$$\int_{-1}^{1} \frac{1}{1+it}\,dt$$
You might reasonably think of using a multiple of $\log(1+it)$ as an antiderivative. Note that this is the complex log so you will need to choose a branch to get a single value. Take care with this choice.