Modeling Project #2

Your experience tells you that a floating object will bob up and down if it is displaced from an equilibrium position. The text discusses the bobbing motion of a floating object in the Introduction to Chapter 4 and in the Extended Problem at the end of Chapter 4. The ideas in the Extended Problem show how this can be modeled as *simple harmonic motion* when displacements from equilibrium are not too big.

Consider the *parabolic trough* described in the Extended Problem. Imagine we are designing a floating platform of this shape. We are asked to avoid bobbing motions with frequencies greater than 2 Hz (Note: 1 Hz=1 per second.) What restrictions does this place on the dimensions of the trough?

Submit a complete report on a modeling process you use to reach a conclusion. Write for an audience of peers in a differential equations course who have seen the material we have covered but have not thought about this particular modeling problem. Assume readers have not read the problem statement. Do not use the wording of problem statement itself.

This project is due on Monday, March 27.

Notes, comments, and hints

1. You can follow steps (a)-(d) described in the text.
2. The problem statement includes the sentence "Assume the body weighs one-half the weight of an equal volume of water." One way to interpret this statement is to assume the *density* of the object is one-half the density of water. This interpretation assumes that the object has a uniform density (i.e., the density is the same at each point of the object). The text's phrasing allows for a non-uniform density so a better interpretation is to assume that the *average density* of the object is one-half the density of water.
3. With the text's definition of the variable y, a push down corresponds to $y > 0$.
4. The text's use of $f(\frac{y}{h})$ in part (d) implies that the expression on the right-hand side can be rewritten so y shows up only in the combination $\frac{y}{h}$.
5. Let $x = \frac{y}{h}$ so $f(\frac{y}{h}) = f(x)$. The Taylor series expansion of f at $x = 0$ is

   $$f(x) = f(0) + f'(0)x + \frac{1}{2}f''(0)x^2 + \dots$$

   Using only up to the first-degree term, we have the approximation

   $$f(x) \approx f(0) + f'(0)x.$$

   Substituting back gives

   $$f(\frac{y}{h}) \approx f(0) + f'(0)\frac{y}{h} = f(0) + \frac{f'(0)}{h}\,y.$$