Name _____

MATH 301 Differential Equations Fall 2005 Exam #3

**Instructions:**

Do your own work. You may consult class notes, the course text, or other books. Give a reference if you use some source other than class notes or the course text. Do not discuss generalities or specifics of the exam with others.

Turn in a complete and concise write up of your work. Show enough detail so that a peer could follow your work (both computations and reasoning). If you are not confident in some result, you will receive more credit if you make a note of this and comment on where you might be going wrong or on alternate approaches you might try.

You can use technology as needed. When you do, give some indication of what you use and how you use it.

Express each result in terms of real-valued functions.

Each problem has a value of 25 points.

The exam is due Thursday, December 1 at 4:00 pm.

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1. Find the general solution of the system $\frac{d}{dt}\vec{y} = A\vec{y}$ where $A = \begin{bmatrix} 9 & 30 & 2 \\ -1 & -8 & -1 \\ -11 & -20 & -4 \end{bmatrix}$.

   Describe the behavior of solutions as $t \to \infty$.

2. Consider the system $\frac{d}{dt}\vec{y} = A\vec{y}$ where $A = \begin{bmatrix} 2\alpha & \beta \\ \beta & 0 \end{bmatrix}$. Find the general solution. Note that the solution may have different forms for different values of $\alpha$ and $\beta$. For each form, give the relevant value or values of the parameters. For each form of solution, make a phase portrait showing solution curves in the $y_1y_2$-plane.

3. Find the general solution of the system $\frac{d}{dt}\vec{y} = A\vec{y} + \vec{g}(t)$ where

$$A = \begin{bmatrix} -7 & 5 \\ -10 & 8 \end{bmatrix} \qquad \text{and} \qquad \vec{g}(t) = e^{-2t}\begin{bmatrix} 5 \\ 2 \end{bmatrix}.$$

4. Consider a chain of radioactive decays $X \to Y \to Z$ where $X$, $Y$, and $Z$ are different radioactive elements. Let $x(t)$, $y(t)$, and $z(t)$ denote the amounts of these elements. Set up and solve an initial value problem to determine $x(t)$, $y(t)$, and $z(t)$ if the process starts with an initial amount $x_0$ of element $X$ and none of elements $Y$ and $Z$. Find expressions for the ratios $y(t)/x(t)$ and $z(t)/x(t)$ and the limits of these ratios as $t \to \infty$. Give an interpretation of these limits.