

The forced undamped spring

An undamped spring-mass system with an external sinusoidal driving force is modeled by

$$mx''(t) + kx(t) = E_0 \sin(\beta t).$$

We solved this in class. The particular solution for the nonhomogeneous problem depends on whether $\beta \neq \omega_0$ or $\beta = \omega_0$ where

$$\omega_0 = \sqrt{\frac{k}{m}}.$$

1. Find the specific solution for the case $\beta \neq \omega_0$ with the initial conditions $x(0) = x_0$ and $x'(0) = v_0$.
2. Find the specific solution for the case $\beta = \omega_0$ with the initial conditions $x(0) = x_0$ and $x'(0) = v_0$.
3. Show that the specific solution for $\beta = \omega_0$ is a limit of the corresponding specific solution for $\beta \neq \omega_0$.