Linear combinations of matrices

1. We have defined each of the following phrases in reference to a set of column vectors in $\mathbb{C}^n$. For each, formulate a new definition in reference to a set of matrices in M_{mn}.

   (a) *linear combination*

   (b) *span*

   (c) *relation of linear dependence*

   (d) *linearly independent* and *linearly dependent*

2. Consider the set
$$S = \left\{ \begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}, \begin{bmatrix} 5 & 2 \\ 2 & 6 \end{bmatrix}, \begin{bmatrix} 4 & 8 \\ 1 & 2 \end{bmatrix} \right\}$$
in M_{22}. For each of the following, determine if the given matrix is in the span of S or not. If so, express the given matrix as a linear combination of the vectors in S.

   (a) $\begin{bmatrix} 4 & 7 \\ 0 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} -3 & -1 \\ 8 & 1 \end{bmatrix}$

3. For each of the following, determine if the given set is linearly independent in M_{22}:

   (a) $S = \left\{ \begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}, \begin{bmatrix} 5 & 2 \\ 2 & 6 \end{bmatrix}, \begin{bmatrix} 4 & 8 \\ 1 & 2 \end{bmatrix} \right\}$

   (b) $S = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$

4. Find a linearly independent spanning set for M_{22}, that is, a linearly independent set S such that $\mathrm{Sp}(S) = M_{22}$.

5. Define $S_{22} = \{A \in M_{22} | A \text{ is symmetric}\}$. Find a linearly independent spanning set for S_{22}, that is, a linearly independent set S such that $\mathrm{Sp}(S) = S_{22}$.

6. Define $U_{22} = \{A \in M_{22} | [A]_{ij} = 0 \text{ if } i > j\}$. Find a linearly independent spanning set for U_{22}, that is, a linearly independent set S such that $\mathrm{Sp}(S) = U_{22}$.

7. Define $AS_{22} = \{A \in M_{22} | A^t = -A\}$. Find a linearly independent spanning set for AS_{22}, that is, a linearly independent set S such that $\mathrm{Sp}(S) = AS_{22}$.