Physics 121MA/Math 122PH
Spring 2005 Project 3 Name _____

Instructions: We encourage you to work with others in a small group on this project. You should write your solution neatly using complete sentences that incorporate all symbolic mathematical expressions into the grammatical structure. Include enough detail to allow a fellow student to reconstruct your work, but you need not show every algebraic or arithmetic step. It is important that you do your own writing, even if you have worked out the details with other people. All graphs should be done carefully on graph paper or drawn by a computer. This project is due in class on Friday, April 29.

Consider the magnetic field due to a straight wire carrying a current I. In cylindrical coordinates with the z-axis running along the wire, this is

$$\vec{B} = \frac{\mu_0 I}{2\pi r}\hat{\theta}.$$

1. Show that the curl of this magnetic field is $\vec{0}$ for $r \neq 0$.
2. Show that $-\frac{\mu_0 I}{2\pi}\arctan\left(\frac{x}{y}\right)$ is a potential function for this field.
3. Evaluate $\oint_C \vec{B} \cdot d\vec{s}$ where C is the circle of radius 1 in the xy-plane centered at $(x, y) = (0, 3)$ by parametrizing the curve.
4. Explain why the result in (c) is exactly the result guaranteed by the Fundamental Theorem of Calculus for Line Integrals.
5. Evaluate $\oint_C \vec{B} \cdot d\vec{s}$ where C is the circle of radius 1 in the xy-plane centered at $(x, y) = (0, 0)$ by parametrizing the curve.
6. Carefully explain how the results of the previous problems are consistent with the Fundamental Theorem of Calculus for Line Integrals and Theorem 27.5 from the text.