Name ______________________

MATH 221A Multivariate Calculus Spring 2003 Final Exam

Instructions: You can work on the problems in any order. Please use just one side of each page and clearly number the problems. You do not need to write answers on the question sheet.

This exam is a tool to help me (and you) assess how well you are learning the course material. As such you should report enough written detail for me to understand how you are thinking about each problem.

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1. (a) Give a component proof for the identity $\vec{a} \cdot (\vec{a} \times \vec{b}) = 0$. (5 points)

   (b) Give a geometric argument for the identity $\vec{a} \cdot (\vec{a} \times \vec{b}) = 0$. (5 points)

2. Consider the vector-output function $\vec{r}(t) = \langle 1 - 4t^2, 6t, t + \frac{1}{3}t^3 \rangle$ parametrizing a curve C. Find a vector tangent to the curve at the point $(-35, 18, 12)$. (12 points)

3. Consider the function $f(x, y) = \dfrac{y}{x^2}$.

   (a) Find the equation of the tangent plane for the input $(2, 3)$. (8 points)

   (b) Find the unit vector in the direction of the greatest slope for f at the input $(2, 3)$. (8 points)

4. Show that $(3, -1)$ is a critical input for the function $f(x, y) = x^2y^3 - 6xy - 9y$ and classify this input as a local minimizer, a local maximizer, or neither. (12 points)

5. Evaluate $\displaystyle\iiint_R x\, dV$ where R is the tetrahedron with vertices at $(0, 0, 0)$, $(0, 0, 2)$, $(4, 0, 2)$, and $(0, 1, 2)$. (12 points)

6. The mass density of a solid sphere of radius R is given, in spherical coordinates, by $\rho \cos \frac{\theta}{2}$. Compute the total mass of the sphere. (12 points)

7. Use Green's theorem to evaluate the line integral $\displaystyle\int_C \vec{F} \cdot d\vec{s}$ where $\vec{F}(x, y) = \langle y^2, x \rangle$ and C is the perimeter of the triangle with vertices at $(2, 0)$, $(2, 3)$ and $(1, 3)$. (12 points)

8. Describe three different ways to compute the formula for the area of a circle of radius R. Each way should use an idea from this course. For each, you need to give enough detail so it is clear precisely what calculation needs to be done, but you do not need to do the calculation. (12 points)