Name ____________________

MATH 221A | Multivariate Calculus | Spring 2003 | Exam #5

Instructions: You can work on the problems in any order. Please use just one side of each page and clearly number the problems. You do not need to write answers on the question sheet.

This exam is a tool to help me (and you) assess how well you are learning the course material. As such you should report enough written detail for me to understand how you are thinking about each problem.

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1. Consider the vector field $\vec{F}(x, y) = \langle y^2, 0 \rangle$.

   (a) Sketch a plot of this vector field. (4 points)

   (b) Use your vector field plot to determine geometrically where in the xy-plane the divergence is positive, where the divergence is negative, and where the divergence is zero. Make your reasoning clear. (4 points)

   (c) Compute the divergence of $\vec{F}(x, y)$. Compare this to your results for (b). (4 points)

   (d) Use your vector field plot to determine geometrically where in the xy-plane the curl is positive, where the curl is negative, and where the curl is zero. Make your reasoning clear. (4 points)

   (e) Compute the curl of $\vec{F}(x, y)$. Compare this to your results for (d). (4 points)

2. For a function $f : \mathbb{R}^2 \to \mathbb{R}$ and a vector field $\vec{F} : \mathbb{R}^2 \to \mathbb{R}^2$, prove the identity (20 points)

$$\vec{\nabla} \cdot (f\vec{F}) = f(\vec{\nabla} \cdot \vec{F}) + (\vec{\nabla} f) \cdot \vec{F}.$$

3. Consider the line integral $\int_C \vec{F} \cdot d\vec{s}$ where $\vec{F}(x, y, z) = \langle z, z, x + y \rangle$ and C is the straight line from $(2, 1, 3)$ to $(5, 0, 1)$.

   (a) Compute $\int_C \vec{F} \cdot d\vec{s}$ by parametrizing the curve. (10 points)

   (b) Compute $\int_C \vec{F} \cdot d\vec{s}$ using the Fundamental Theorem for Line Integrals. (10 points)

4. Use Green's Theorem to evaluate the line integral $\oint_C \vec{F} \cdot d\vec{s}$ where $\vec{F}(x, y) = \langle x^3 + 3y^2, x^4 \rangle$ and C is the perimeter of the square $[0, 1] \times [0, 2]$ traversed clockwise. (20 points)

5. Let C be a simple closed curve in the plane that encloses a region with area equal to 15 square units. Evaluate $\oint_C \langle 5y, -3x \rangle \cdot d\vec{s}$. (20 points)