Name _____

MATH 221A Multivariate Calculus Spring 2003 Exam #4

Instructions: You can work on the problems in any order. Please use just one side of each page and clearly number the problems. You do not need to write answers on the question sheet.

This exam is a tool to help me (and you) assess how well you are learning the course material. As such you should report enough written detail for me to understand how you are thinking about each problem.

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1. Set up and evaluate an interated integral (or integrals) to compute the value of the double integral of the function $f(x, y) = x^2 y$ over the region R in the xy-plane bounded below by $y = x^3$ and above by $y = \sqrt{32x}$. (20 points)

2. Set up, but do not evaluate, an iterated integral (or integrals) that gives the value of the triple integral of the function $f(x, y, z) = xy^2 + z^2$ over the solid region bounded by the planes $x = 0$, $y = 0$, $y = 5$, $z = 0$, and $z = 4 - 2x$. (20 points)

3. Set up, but do not evaluate, an iterated integral (or integrals) that gives the volume of the solid region that is above the xy-plane and inside both the sphere of radius 10 centered at the origin and the cone $z = \sqrt{x^2 + y^2}$. Express your result entirely in terms of a single coordinate system (of your choice). (20 points)

4. Set up, but do not evaluate, an iterated integral (or integrals) that gives the total mass of the top half of the solid sphere of radius R centered at the origin with a mass density proportional to the square of the height above the xy-plane. Express your result entirely in terms of a single coordinate system (of your choice). (20 points)

5. (a) Give a geometric argument for why the correct area element in polar coordinates is $dA = r\,dr\,d\theta$. Include at least one relevant figure. (10 points)

   (b) Give a geometric argument for why the correct volume element in spherical coordinates is $dV = \rho^2 \sin\phi\,d\rho\,d\phi\,d\theta$. Include at least one relevant figure. (10 points)