Name ____________________

MATH 221A | Multivariate Calculus | Spring 2003 | Exam #3

Instructions: You can work on the problems in any order. Please use just one side of each page and clearly number the problems. You do not need to write answers on the question sheet.

This exam is a tool to help me (and you) assess how well you are learning the course material. As such you should report enough written detail for me to understand how you are thinking about each problem.

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1. Consider the function $f(x, y) = \frac{1}{3}x^3 + 5xy^2 + yz$.

   (a) Find the maximum rate of change in f at the point $(3, -2, 1)$. (8 points)

   (b) Compute the directional derivative $D_{\hat{u}}f$ at the point $(3, -2, 1)$ in the direction of the point $(0, 2, 7)$. (8 points)

   (c) Give an interpretation of the value you compute in (b). (4 points)

2. The figure on the accompanying sheet shows level curves for a function $f : \mathbb{R}^2 \to \mathbb{R}$.

   (a) For each of the points A and B indicated on the plot, estimate the magnitude of the gradient vector. (8 points)

   (b) Draw a gradient vector at each of the two points indicated on the plot. Use your estimates from (a) to draw vectors with appropriate relative lengths. (8 points)

3. Consider the function $f(x, y) = 18x^2 + y^2 - 2x^2y + 7$.

   (a) Show that $(0, 0)$ and $(-3, 9)$ are critical inputs for f. (8 points)

   (b) Determine if $(0, 0)$ is a local minimizer, a local maximizer, or neither. (5 points)

   (c) Determine if $(-3, 9)$ is a local minimizer, a local maximizer, or neither. (5 points)

4. Find the global minimum and global maximum for $f(x, y) = xy - x - y^2$ on the closed triangular region with vertices at $(0, 0)$, $(3, 3)$, and $(-3, 3)$. (18 points)

5. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be differentiable. For each of the following, sketch a plot that shows level curves and gradient vectors in some small region of the xy-plane centered at a point (x_0, y_0) where (4 points each)

   (a) (x_0, y_0) is a typical local minimizer.

   (b) (x_0, y_0) is a typical local maximizer.

   (c) (x_0, y_0) is a typical "saddlizer."

6. The temperature in a certain room is given by the function $f(x, y, z) = xyz$. A fly cruises through the room on the path $\vec{r}(t) = \langle t, t^2, t^3 \rangle$ where t is time. What rate of change in temperature with respect to time does the fly experience at time $t = 2$? For units, assume temperature is measured in $^\circ C$, length is measured in meters, and time is measured in seconds. (16 points)