Name _____

MATH 221A Multivariate Calculus Spring 2003 Exam #2

Instructions: You can work on the problems in any order. Please use just one side of each page and clearly number the problems. You do not need to write answers on the question sheet.

This exam is a tool to help me (and you) assess how well you are learning the course material. As such you should report enough written detail for me to understand how you are thinking about each problem.

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1. The acceleration of a particle is given by $\vec{a}(t) = \cos t\,\hat{\imath} + \sin t\,\hat{\jmath} + 3\,\hat{k}$. (12 points)

   (a) Find the velocity function $\vec{v}(t)$ for the particle given that $\vec{v}(0) = \hat{\imath} + \hat{\jmath}$.

   (b) Find the position function $\vec{r}(t)$ for the particle given that $\vec{r}(0) = \hat{\jmath} + 2\hat{k}$.

2. Consider the two curves given by the vector-output functions $\vec{r}(t) = \langle t, t^2, t^3 \rangle$ and $\vec{p}(s) = \langle s + 3, 2s^2 + 2, 9 - s^4 \rangle$.

   (a) Confirm that each curve contains the point $(2, 4, 8)$. (4 points)

   (b) Find the angle between the curves at the point of intersection $(2, 4, 8)$. (8 points)

3. Consider the function $f(x, y) = \begin{cases} \dfrac{x^3 y}{x^4 + y^4} & (x, y) \neq (0, 0) \\ A & (x, y) = (0, 0). \end{cases}$

   Show that no value of A can be chosen so that $f(x, y)$ is continuous at $(0, 0)$. (10 points)

4. Compute the first and second partial derivatives of $f(x, y) = xy^2 e^{x+y}$. (10 points)

5. Consider a function $f : \mathbb{R}^3 \to \mathbb{R}$ for which we know the following information:

   $$f(2, 4, -1) = 5, \quad \frac{\partial f}{\partial x}(2, 4, -1) = 2, \quad \frac{\partial f}{\partial y}(2, 4, -1) = -3, \quad \text{and} \quad \frac{\partial f}{\partial z}(2, 4, -1) = 6.$$

   Estimate $f(1.8, 4.1, -0.7)$. (10 points)

6. Find the equation of the tangent plane for $f(x, y) = x^3 y^2$ at the point with $(x, y) = (1, 2)$. (12 points)

7. Consider a function of the type $f : \mathbb{R}^4 \to \mathbb{R}$ with outputs denoted $f(p, q, r, s)$.

   (a) Write the definition (in terms of a difference quotient) of $\dfrac{\partial f}{\partial r}$ and give an interpretation of $\dfrac{\partial f}{\partial r}$. (5 points)

   (b) Compute $\dfrac{\partial f}{\partial r}$ for $f(p, q, r, s) = pr^2 \sin(qrs)$. (5 points)

8. The accompanying plot shows level curves for a function $f(x, y)$.

   (a) Describe the graph of this function. Give coordinates (x, y, z) for the top of each peak and the bottom of each pit. (8 points)

   (b) Estimate the partial derivatives of the function $f(x, y)$ at the point $(3, -3)$. (6 points)

9. The gravitational force F on a particle of mass m at a distance r from the center of the earth is given by the formula

$$F = k\frac{m}{r^2}$$

   where k is a constant. Relate the percentage change in F to percentage changes in m and r. (10 points)