Name ____________________

MATH 221A | Multivariate Calculus | Spring 2003 | Exam #1

Instructions: You can work on the problems in any order. Please use just one side of each page and clearly number the problems. You do not need to write answers on the question sheet.

This exam is a tool to help me (and you) assess how well you are learning the course material. As such you should report enough written detail for me to understand how you are thinking about each problem.

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1. For each of the following, compute the given expression for the vectors $\vec{u} = 2\hat{\imath} - 5\hat{\jmath} - 3\hat{k}$ and $\vec{v} = \langle 4, 5, -2 \rangle$. (4 points each)

   (a) $4\vec{u} - 3\vec{v}$ (b) $\vec{u} \cdot \vec{v}$ (c) $\vec{u} \times \vec{v}$

2. For each of the following, determine if the given expression is a scalar, a vector, or undefined. (2 points each)

   (a) $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d})$
   (b) $(\vec{a} \times \vec{b}) \times (\vec{c} \cdot \vec{d})$
   (c) $[(\vec{a} \cdot \vec{b})\vec{c}] \cdot \vec{d}$
   (d) $[(\vec{a} \times \vec{b}) \times \vec{c}] \times \vec{d}$

3. Find the equation of the plane that contains the points $(2, 0, 4)$, $(1, -3, 5)$, and $(3, 5, 7)$. (10 points)

4. Consider the plane given by the equation $2x + 8y - z = 4$ and the line given by the parametric equation $\vec{r} = \langle 2 + 4t, 3t, -1 - t \rangle$.

   (a) Find the point at which the plane and line intersect. (6 points)
   (b) Find the angle between a normal for the plane and a direction vector for the line. (8 points)

5. Consider the two vectors $\vec{u} = \langle 2, 1, 1 \rangle$ and $\vec{v} = \langle 9, -3, 1 \rangle$

   (a) Find the scalar projection of $\vec{u}$ in the direction of $\vec{v}$. (6 points)
   (b) Find a unit vector that is perpendicular to both $\vec{u}$ and $\vec{v}$. (6 points)

6. Suppose that the three nonzero vectors $\vec{u}$, $\vec{v}$, and $\vec{w}$ are mutually perpendicular.

   (a) Give a *geometric* argument to show that $\vec{u} \times (\vec{v} \times \vec{w}) = 0$. (6 points)
   (b) Given an *algebraic* argument to show that $\vec{u} \times (\vec{v} \times \vec{w}) = 0$.
   Hints: Think about using identities rather than working with components explicitly. You can use the fact that if two vectors are perpendicular, then the dot product of the two is zero. (6 points)

7. Consider the triangle with vertices at $(3, -4, 5)$, $(2, 1, -1)$ and $(3, 4, 2)$.

   (a) Calculate the angle at the vertex $(3, -4, 5)$. (6 points)

   (b) Calculuate the area of the triangle. (6 points)

8. Calculate the volume of the parallelepiped with edges defined by the vectors $\langle 8, 0, 9\rangle$, $\langle 3, 6, -5\rangle$, and $\langle 5, 2, 0\rangle$. (10 points)

9. Find a vector output function $\vec{r}(t)$ that traces out the ellipse $\dfrac{x^2}{9} + \dfrac{y^2}{4} = 1$ in the counter-clockwise direction as t increases starting at the point $(0, 2)$ for $t = 0$. (10 points)

Identities

$$\|\vec{a} \times \vec{b}\|^2 = \|\vec{a}\|^2 \|\vec{b}\|^2 - (\vec{a} \cdot \vec{b})^2$$

$$\vec{a} \times (\vec{b} \times \vec{c}) = \vec{b}(\vec{a} \cdot \vec{c}) - \vec{c}(\vec{a} \cdot \vec{b})$$

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = \vec{c} \cdot (\vec{a} \times \vec{b}) = \vec{b} \cdot (\vec{c} \times \vec{a})$$