

**Instructions:** You can work on the problems in any order. Please use just one side of each page and clearly number the problems. You do not need to write answers on the question sheet.

This exam is a tool to help me (and you) assess how well you are learning the course material. As such you should report enough written detail for me to understand how you are thinking about each problem.

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1. Consider the limit statement  $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$ .

(a) Explain the meaning of this limit statement. (4 points)

(b) Give an argument or evidence to convince someone that this limit statement is correct. (4 points)

2. Find values for  $a$  and  $b$  so that the function

$$f(x) = \begin{cases} x^2 + 7 & \text{if } x < 3 \\ b & \text{if } x = 3 \\ ax + 2 & \text{if } x > 3 \end{cases}$$

is continuous for all  $x$ . (8 points)

3. (a) Give a definition, equivalent to that in the text, for *the derivative of a function*. (4 points)

(b) Give *two* distinct interpretations of derivative. (6 points)

4. Differentiate each of the following functions. (5 points each)

(a)  $g(t) = t^3 - 3t^2 + 4t + 7$

(b)  $f(x) = \cos(x^2) + xe^x$

5. Find the equation of the tangent line for the function  $f(x) = \frac{3}{x^2}$  at the point with  $x = 5$ . (8 points)

6. Find the absolute minimum and absolute maximum for the function  $f(x) = 2x^3 - 12x^2 + 5$  on the interval  $[-1, 7]$ . (8 points)

7. A rectangle is inscribed in a semicircle with one edge of the rectangle along the diameter of the semicircle. The diameter of the semicircle is 10 inches. Find the dimensions of the rectangle with largest area. (8 points)
8. A car moves along a straight road with velocity given by  $v(t) = 4t^{3/2}$  (measured in meters per second).
- (a) Find the acceleration of the car for  $t = 9$  seconds. (5 points)
- (b) Find the position of the car for  $t = 9$  seconds assuming the car is at position  $s = 3$  meters for  $t = 1$  second. (5 points)
9. Find the slope of the tangent line to the curve given by the equation  $xy + x^2 + y^3 = 7$  at the point  $(2, 1)$ . (8 points)
10. A spot of light from a searchlight is sweeping horizontally along a straight wall (in search of several calculus students who have broken out of Thompson Hall). The searchlight is located 30 meters from the wall directly in front of a door. The searchlight is rotated on its mount at a rate of 0.5 radians per second. How fast is the spot of light moving along the wall at the instant the spot is 20 meters from the door? (8 points)
11. Evaluate  $\int (e^x + x^3 + \sin x) dx$ . (6 points)
12. Consider the function  $f(x) = \frac{1}{x}$  for the interval  $[1, 3]$ . Estimate the area of the region between the graph of this function and the  $x$ -axis using a sum of the area of 6 rectangles constructed with right endpoints. Show enough detail so that it is clear how you arrive at your result. (8 points)