Name _____

MATH 121C Calculus and Analytic Geometry I Spring 2001 Exam #4

Instructions: You can work on the problems in any order. Please use just one side of each page and clearly number the problems. You do not need to write answers on the question sheet.

This exam is a tool to help me (and you) assess how well you are learning the course material. As such you should report enough written detail for me to understand how you are thinking about each problem.

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1. Consider the function $f(x) = \dfrac{x^2 - 8x - 20}{x^2}$.

   (a) Analyze any vertical asymptotes. (4 points)

   (b) Analyze any horizontal asymptotes. (4 points)

   (c) Determine the inputs for which the function is positive and the inputs for which the function is negative. (4 points)

   (d) Determine the inputs for which the function is increasing and the inputs for which the function is decreasing. (4 points)

   (e) Determine the inputs for which the function is concave up and the inputs for which the function is concave down. (4 points)

   (f) Sketch a graph of the function and label any asymptotes, zeros, local minima, local maxima, and inflection points. (4 points)

2. A rectangular window is be built with an area of 10 square feet. The material for the bottom edge of the window frame costs 30 cents per foot and the material for the side and top edges of the window frame costs 20 cents per foot. Find the dimensions that minimize the total cost of the window frame. (10 points)

3. Analyze each of the following limits. (9 points each)

   (a) $\lim\limits_{x \to 0} \dfrac{\sin x - x}{x^3}$ (b) $\lim\limits_{x \to 0} x^{2x}$

4. Consider the function $f(x) = 5x$ for the interval $[1, 3]$. Compute the area of the region between the graph of this function and the x-axis using a sum of the area of n rectangles in the limit as $n \to \infty$. (8 points)

5. Consider the function $f(x) = e^x$ for the interval $[0, 4]$. Estimate the area of the region between the graph of this function and the x-axis using a sum of the area of 5 rectangles constructed with right endpoints. Show enough detail so that it is clear how you arrive at your result. (8 points)

6. Evaluate each of the following indefinite integrals. (8 points each)

(a) $\int (x^2 + 3x - 6)\, dx$

(b) $\int (\cos t - \frac{1}{t^2})\, dt$

7. Let $K(t)$ be the calculus knowledge (measured in units called *smarts*) that a student has after t hours of studying. Suppose a particular student starts with 12 smarts and gains calculus knowledge at a rate of $6\sqrt{t}$ smarts per hour. How much calculus knowledge will the student have after 3 hours of studying? (10 points)

8. Give a definition, equivalent to that in the text, for the phrase *the function $F(x)$ is an antiderivative of the function $f(x)$.* (6 points)

Some sum facts

$$\sum_{k=1}^{n} 1 = n \qquad \sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}$$