

Instructions: You can work on the problems in any order. Please use just one side of each page and clearly number the problems. You do not need to write answers on the question sheet.

This exam is a tool to help me (and you) assess how well you are learning the course material. As such you should report enough written detail for me to understand how you are thinking about each problem.

1. Give a definition, equivalent to that in the text, for each of the following terms. (6 points each)

(a) *function*

(b) *limit of a function*

2. For each of the following, solve the given equation. (6 points each)

(a) $\cos^2 x - \frac{1}{2} \cos x = 0$ for x in $[0, 2\pi)$

(b) $\log(2x + 3) - \log(x) = \log(x)$

3. Simplify $\cos(\tan^{-1} x)$. That is, rewrite $\cos(\tan^{-1} x)$ as an algebraic expression. (6 points)

4. Given that $(f \circ g)(x) = (x + 5)^{3/2}$ and $f(x) = \sqrt{x}$, determine $g(x)$. (6 points)

5. The population of a bacteria colony is growing exponentially so that $P(t) = Ae^{kt}$ where t is time measured in hours, $P(t)$ is the population, measured in thousands, at time t , and A and k are constants. The population at time $t = 0$ is measured to be 20 thousand and measured again at time $t = 10$ hours to be 50 thousand. Find the population at time $t = 25$ hours. (6 points)

6. Consider the function $f(x) = \begin{cases} 15 - 2x & \text{if } 0 \leq x \leq 3, \\ x - 1 & \text{if } 3 < x < 6. \end{cases}$ (4 points each)

(a) Sketch a graph of this function.

(b) Determine the domain and range of this function.

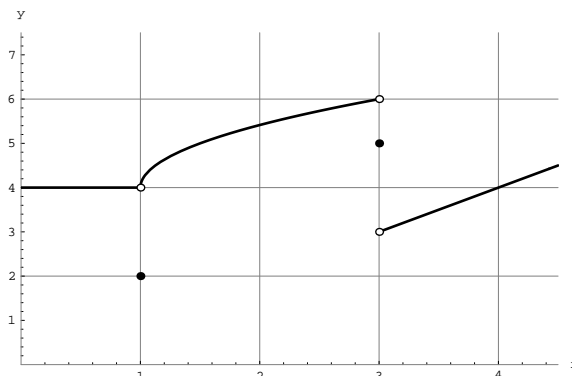
(c) Determine if this function has an inverse function. If this function does have an inverse, find the inverse function. If not, explain why.

7. Analyze $\lim_{x \rightarrow 0} \frac{\tan x - x}{x^3}$ using a table of input/output pairs. (7 points)

8. The plot below shows the graph of a function f . Use the graph to analyze each of the following limits. Explain how you arrive at your conclusions. (6 points each)

(a) $\lim_{x \rightarrow 1} f(x)$

(b) $\lim_{x \rightarrow 3} f(x)$



9. For each of the following, evaluate the limit using techniques that give an exact result if possible. Show enough details to make your methods clear to a reader. (7 points each)

(a) $\lim_{x \rightarrow 16} \frac{\sqrt{x} - 4}{x - 16}$

(b) $\lim_{x \rightarrow 0} \frac{x}{\sin x}$

(c) $\lim_{x \rightarrow 2} f(x)$ where $f(x) = \begin{cases} 2x + 5 & \text{if } x < 2, \\ 5 & \text{if } x = 2 \\ x^2 + 4 & \text{if } x > 2. \end{cases}$

10. The table to the right gives input/output pairs for a function f . Is it possible that the limit of this function as x approaches 2 is *not* equal to 5. Explain how you reach your conclusion. (6 points)

x	$f(x)$
1.9	5
1.99	5
1.999	5
2.001	5
2.01	5
2.1	5

A few trigonometric identities

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta = 2 \cos^2 \theta - 1 = 1 - 2 \sin^2 \theta$$

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

$$\sin^2 \theta = \frac{1}{2}(1 - \cos 2\theta)$$

$$\cos^2 \theta = \frac{1}{2}(1 + \cos 2\theta)$$

$$\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$$

$$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$$

$$\cos \frac{1}{2}\theta = \pm \sqrt{\frac{1 + \cos \theta}{2}}$$

$$\sin \frac{1}{2}\theta = \pm \sqrt{\frac{1 - \cos \theta}{2}}$$