Name ________________________________

MATH 121C Calculus and Analytic Geometry I Spring 2001 Exam #3

Instructions: You can work on the problems in any order. Please use just one side of each page and clearly number the problems. You do not need to write answers on the question sheet.

This exam is a tool to help me (and you) assess how well you are learning the course material. As such you should report enough written detail for me to understand how you are thinking about each problem.

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1. Find the slope of the line tangent to the curve with equation $x^3y - xy^2 = 30$ at the point $(x, y) = (4, 1)$. (8 points)

2. Find the absolute minimum and the absolute maximum of the function $f(x) = x^4 - 4x^2 + 2$ on the interval $[-3, 2]$. (10 points)

3. Give a definition, equivalent to that in the text, for each of the following terms. (6 points each)
   (a) *x_0 is a (first-order) critical input for the function f*
   (b) *$(x_0, f(x_0))$ is an inflection point for the function f*

4. (a) Give a statement of the Mean Value Theorem equivalent to that used in the text. (8 points)
   (b) Use the Mean Value Theorem to argue that *no* differentiable function can have all of the following three properties: $f(-1) = 3$, $f(4) = 13$, and $f'(x) \geq 5$ for all x. (6 points)

5. The volume V of a cube is to be measured and the length $L = \sqrt[3]{V}$ of the cube is to be computed. Use the linear approximation to find an approximate relationship between the percentage error $\frac{\Delta V}{V_0}$ in the measured quantity and the percentage error $\frac{\Delta L}{L_0}$ in the computed quantity. (8 points)

6. Consider the function $f(x) = xe^{-Ax}$ where A is a positive constant.
   (a) Determine the set of inputs for which f is positive and the set of inputs for which f is negative. (6 points)
   (b) Determine the set of inputs for which f is increasing and the set of inputs for which f is decreasing. (6 points)
   (c) Determine the set of inputs for which f is concave up and the set of inputs for which f is concave down. (6 points)
   (d) Sketch a graph of f and, on the plot, label the essential features of the graph such as any relative minima, relative maxima, or inflection points. (6 points)

7. Show how to find the derivative of the inverse tangent function $y = \tan^{-1}(x)$ using the fact that $\dfrac{d}{du}[\tan(u)] = \sec^2(u)$. (6 points)

8. A spot of light from a searchlight is sweeping horizontally along a straight wall (in search of several calculus students who have broken out of Thompson Hall). The searchlight is located 30 meters from the wall directly in front of a door. The searchlight is rotated on its mount at a rate of 0.5 radians per second. How fast is the spot of light moving along the wall at the instant the spot is 20 meters from the door? (10 points)

9. The plot below shows the graph of a function f and the tangent line for f at the point $(a, f(a))$. For each of the following, give an expression using the inputs a and x and outputs of the function f and its derivative f'. (2 points each)

   (a) the length of the segment AB

   (b) the length of the segment BC

   (c) the length of the segment AC

   (d) the length of the segment AD