

Instructions: You can work on the problems in any order. Please use just one side of each page and clearly number the problems. You do not need to write answers on the question sheet.

This exam is a tool to help me (and you) assess how well you are learning the course material. As such you should report enough written detail for me to understand how you are thinking about each problem.

1. Give a definition, equivalent to that in the text, for each of the following terms. (6 points each)

- (a) a function f is continuous for the input a
(b) a function f is differentiable for the input a

2. Consider the function $f(x) = \begin{cases} x^2 & \text{if } x < 3 \\ 3\sqrt{x+6} & \text{if } x \geq 3. \end{cases}$

- (a) Determine if f is continuous for the input $x = 3$. Explain how you reach your conclusion. (5 points)
(b) Determine if f is differentiable for the input $x = 3$. Explain how you reach your conclusion. (5 points)

3. Differentiate each of the following functions. (6 points each)

- (a) $f(x) = 3x^4 + 12x^3 + 3x^2 - 5x + 2$ (d) $h(x) = x^2e^{-x}$
(b) $f(x) = \frac{3 \sin x}{x^2}$ (e) $f(x) = \sqrt{x^3 + 4x}$
(c) $g(t) = \sin(\cos t)$ (f) $f(x) = (x^2 + 2x)^3 \ln x$

4. Use the definition of derivative to explain why $f'(a)$ can be interpreted as the slope of the tangent line for the graph of the function f at the point $(a, f(a))$ (6 points)

5. Find the equation of the tangent line for $f(x) = x^4 + 5x + 2$ for $x = 2$. (8 points)

6. Let t represent time (in minutes) and $V(t)$ represent the volume (in gallons) of water in a tank. For the time interval $[0, 6]$, the volume is found to be given by $V(t) = 3t^2 + 6 \sin(\pi t)$. Find the rate of change in the volume for $t = 2$. (8 points)

7. The position of a car moving along a straight road is given by $s(t) = \frac{5}{3}t^3 - 20t^2 + 60t + 10$ with position measured in miles and time measured in hours. (4 points each)

- (a) Find the velocity of the car at time $t = 1$ hr.
(b) Find the acceleration of the trolley at time $t = 2$ hr.
(c) Find all of the times at which the car changes direction.

8. The plots on the left below show the graphs of four functions. The plots on the right show the graphs of the derivative functions for the four functions. Match each function with its derivative. (8 points)

